Recommendations in Signed Social Networks

Jiliang Tang[‡], Charu Aggarwal[#], and Huan Liu[†]
[†]Computer Science & Engineering, Arizona State University, Tempe, AZ, USA
[‡]Yahoo Labs, Sunnyvalue, CA, USA
[#] IBM T. J. Watson Research Center, Yorktown Heights, NY, USA
[‡]jlt@yahoo-inc.com, [#]charu@us.ibm.com, [†]huan.liu@asu.edu

ABSTRACT

Recommender systems play a crucial role in mitigating the information overload problem in social media by suggesting relevant information to users. The popularity of pervasively available social activities for social media users has encouraged a large body of literature on exploiting social networks for recommendation. The vast majority of these systems focus on unsigned social networks (or social networks with only positive links), while little work exists for signed social networks (or social networks with positive and negative links). The availability of negative links in signed social networks presents both challenges and opportunities in the recommendation process. We provide a principled and mathematical approach to exploit signed social networks for recommendation, and propose a model, RecSSN, to leverage positive and negative links in signed social networks. Empirical results on real-world datasets demonstrate the e ectiveness of the proposed framework. We also perform further experiments to explicitly understand the e ect of signed networks in RecSSN.

General Terms

Algorithms

Keywords

Social Recommendation; Signed Networks; Negative Links

1. INTRODUCTION

Recommender systems play a crucial role in helping online users collect relevant information by suggesting information of potential interest. The increasing popularity of social media allows online users to participate in online activities in a pervasive way. These social networks provide independent sources of recommendation and unleash previously unknown potentials of recommender systems. The exploitation of social networks for recommendation has attracted increasing interest in recent years [3, 16, 6, 17, 7]. Existing social recommender systems can be roughly categorized into memory-based systems and model-based systems [32]. The vast majority of these systems focus on unsigned social networks (or social networks with only positive links); however, social networks in social media can contain both positive and negative links. Examples of these signed social networks include Epinions[1] with trust and distrust links, and Slashdot[2] with friend and foe links. Such networks provide a much richer source of information than what is exploited by the current systems [39, 13, 1, 5].

Experience with real-world social systems such as Epinions and eBay suggests that negative links in signed social networks are at least as important as positive links [4]. Negative links tend to be more noticeable and credible, and weighed more than positive links of a similar magnitude [21, 2]; therefore, they can be critical in various analytical tasks. For example, negative links add a signi cant amount of knowledge than that embedded only in positive links [12, 29], and a small number of negative links can improve the performance of positive link prediction remarkably [4, 14]. Evidence from recent achievements in signed social network analysis suggests that negative links may also be potentially helpful in recommender systems. However, negative links exhibit very di erent properties from positive links [28, 33]; hence, recommendation with signed social networks cannot be successfully carried out by simply extending recommender systems with unsigned social networks in a straightforward way. For example, most existing recommender systems with unsigned social networks assume that a user's preference is similar to or in uenced by their friends (or positive links) according to homophily [22] and social in uence [19]. Such assumptions are not applicable in signed social networks [33]. This makes the recommendation problem more challenging in the signed network scenario.

In this paper, we study the problem of recommendation with signed social networks, in the context of (i) exploiting positive and negative links in signed social networks; and (ii) modeling them mathematically for recommendation. In order to address these two challenges, we propose the RecSSN framework, in which the primary contributions are as follows:

- We provide a principled approach to mathematically exploit signed social networks for recommendation;

WWW 2016, April 11–15, 2016, Montréal, Québec, Canada.
ACM 978-1-4503-4143-1/16/04.
http://dx.doi.org/10.1145/2872427.2882971.

[1]http://www.epinions.com/
[2]http://slashdot.org/

- We propose a novel recommendation framework, denoted by RecSSN, which captures both positive and negative links in signed social networks; and
- We evaluate the proposed framework in real-world social media datasets to understand the effectiveness and mechanisms of the proposed framework.

The remainder of this paper is organized as follows. In Section 2, we formally define the problem of recommendation with signed social networks. We describe the datasets and perform preliminary data analysis on these datasets in Section 3. In Section 4, we provide approaches to model signed social networks and introduce details about the proposed RecSSN framework with an optimization algorithm. Section 5 presents experimental results with discussions. Section 6 briefly reviews related work. Finally, Section 7 concludes with future work.

2. PROBLEM STATEMENT

Let $\mathcal{U} = \{u_1, u_2, \ldots, u_N\}$ be the set of users and $\mathcal{V} = \{v_1, v_2, \ldots, v_m\}$ be the set of items where N and m are the numbers of users and items, respectively. We assume that $\mathbf{R} \in \mathbb{R}^{N \times m}$ is the user-item matrix where $\mathbf{R}_{ij}$ denotes an observed score from u_i to v_j and we set $\mathbf{R}_{ij} = 0$ if the score from u_i to v_j is missing. Note that in different recommender systems, the score has different meanings. For example, in rating systems such as Epinions and Netflix, scores denote rating scores from users to items; in tagging systems such as Slashdot and Flikcr, scores indicate whether users are associated with tags.

For the problem of recommendation with signed social networks, signed social networks among users are also available in addition to the user-item matrix $\mathbf{R}$. A signed social network $\mathcal{G}$ can be decomposed into a positive component $\mathcal{G}_p$ and a negative component $\mathcal{G}_n$. Let $\mathbf{A}^p \in \mathbb{R}^{N \times N}$ be the adjacency matrix of $\mathcal{G}_p$ where $\mathbf{A}^p_{ij} = 1$ if u_i has a positive link to u_j and $\mathbf{A}^p_{ij} = 0$ otherwise. Similarly, $\mathbf{A}^n \in \mathbb{R}^{N \times N}$ denotes the adjacency matrix of $\mathcal{G}_n$ where $\mathbf{A}^n_{ij} = 1$ if u_i has a negative link to u_j, and $\mathbf{A}^n_{ij} = 0$ otherwise. Note that we only consider links with a binary weight $\{0, 1\}$ in this paper although the generalization of the proposed framework to links with continuous weights is straightforward.

With the aforementioned notations and definitions, the problem of recommendation with a signed social network can be formally stated as follows:

Given observed values in $\mathbf{R}$ and a signed social network $\mathcal{G}$ with positive links $\mathbf{A}^p$, and negative links $\mathbf{A}^n$, the problem of recommendation with a signed social network aims to infer missing values in $\mathbf{R}$.

3. DATA ANALYSIS

Because recommendation with unsigned networks strongly depends on the finding that users are likely to share similar preferences with their friends [32], it is natural to explore similar findings of signed social networks for recommendation. Such an understanding lays the groundwork for a meaningful recommendation framework with signed social networks. In this section, we first introduce the datasets and then perform preliminary data analysis to understand the impact of signed social networks on recommendation.

Table 1: Statistics of the Epinions and Slashdot datasets.

	Epinions	Slashdot
# of Users	18,210	11,868
# of Items	41,089	27,942
# of Positive Links	358,985	290,719
# of Negative Links	75,091	67,108
Density of User-item Matrix	8.42e-4	1.20e-3
# of Users with Negative Links	11,598	7,837

3.1 Datasets

For the purpose of this study, we collected two datasets from Epinions and Slashdot. Some details about these datasets are described below.

Epinions is a popular product review site. Users in Epinions can create both positive (trust) and negative (distrust) links to other users, which results in a signed network $\mathcal{G}$. They can also rate various products with scores ranging from 1 to 5. Therefore, if u_i rates v_j, $\mathbf{R}_{ij}$ is the rating score, and $\mathbf{R}_{ij} = 0$ otherwise.

Slashdot is a technology news platform. Users in Slashdot can create friend (positive) and foe (negative) links to other users, which results in the signed network $\mathcal{G}$. They also can specify tags associated with them. Therefore if v_j is associated with u_i, $\mathbf{R}_{ij} = 1$, and $\mathbf{R}_{ij} = 0$ otherwise.

Some additional preprocessing was performed on these two datasets by filtering users without any positive and negative links, or with few non-zero entities in the user-item matrix $\mathbf{R}$. A number of key statistics of these datasets are illustrated in Table 1[3]. It is evident from these statistics that (i) positive links are denser than negative links in signed social networks; (ii) not all users in signed social networks have negative links; and (iii) the user-item matrix is very sparse.

3.2 An Analysis of Signed Social Networks

Previous studies suggest that users in unsigned social networks are likely to share similar preferences with their friends, which serves as the basis of most existing recommender systems with unsigned social networks [35]. In this subsection, we investigate similar preference properties of users in signed social networks.

Let p_i, n_i and r_i denote the number of users with positive, negative and no links with u_i. We construct three circles for each user u_i with the same size of $\min(p_i, n_i, r_i)$. These circles correspond to (i) a friend circle $\mathcal{FR}_i$ including randomly selected users who have positive links with u_i; (ii) a foe circle $\mathcal{FO}_i$ containing randomly selected users who have negative links with u_i; and (iii) a random circle $\mathcal{RA}_i$ including randomly selected users who have no links with u_i. Similar to the study of preference properties of users in unsigned social networks [35], we investigate preference properties of users in signed social networks by investigating similarities between users and their circles. We will use the friend circle as an example to illustrate how we perform these investigations.

Let $\mathcal{F}_k^{ip}$ be the set of users from $\mathcal{FR}_i$ from whom we observe scores to the item v_k as

$$\mathcal{F}_k^{ip} = \{u_j | u_j \in \mathcal{FR}_i \wedge \mathbf{R}_{jk} \neq 0\} \quad (1)$$

[3]We will make these two datasets publicly available via http://jiliang.xyz/signed.html

Table 2: Average similarities between users and their circles.

Epinions			
	CI	COSINE	CI-COSINE
“Friend” Circles ($\mathbf{s}^p$)	6.4520	0.0292	0.4954
“Foe” Circles ($\mathbf{s}^n$)	2.0808	0.0167	0.3811
Random Circles ($\mathbf{s}^r$)	1.2014	0.0092	0.2497
Slashdot			
	CI	COSINE	CI-COSINE
“Friend” Circles ($\mathbf{s}^p$)	8.5517	0.0456	0.5141
“Foe” Circles ($\mathbf{s}^n$)	2.5035	0.0206	0.4329
Random Circles ($\mathbf{s}^r$)	1.7151	0.0129	0.3226

Then, the average score of FR_i to the k-th item $\mathbf{R}_k^{ip}$ is calculated as follows:

$$\mathbf{R}_k^{ip} = \begin{cases} \frac{\sum_{u_j \in \mathcal{F}_k^{ip}} \mathbf{R}_{jk}}{|\mathcal{F}_k^{ip}|} & \text{for } F_k^{ip} \neq \emptyset, \\ 0 & \text{otherwise.} \end{cases} \quad (2)$$

With $\mathbf{R}_k^{ip}$, we can calculate the similarity s_i^p between u_i and her friend circle FR_i. In this work, we investigate three ways of calculating s_i^p as follows:

CI: We compute s_i^p as the number of common items scored by both u_i and his/her friend circle FR_i as:

$$s_i^p = |I_i^p|, \quad I_i^p = \{v_j | \mathbf{R}_{ij} \neq 0 \wedge \mathbf{R}_j^{ip} \neq 0\} \quad (3)$$

COSINE: The term s_i^p is calculated as cosine similarity of scores between u_i and FR_i over all items as:

$$s_i^p = \frac{\sum_{v_j} \mathbf{R}_{ij} \cdot \mathbf{R}_j^{ip}}{\sqrt{\sum_{v_j} \mathbf{R}_{ij}^2} \sqrt{\sum_{v_j} (\mathbf{R}_j^{ip})^2}}, \quad (4)$$

CI-COSINE: Different from COSINE, CI-COSINE computes the cosine similarity over common items I_i^p as:

$$s_i^p = \frac{\sum_{v_j \in \mathcal{I}_i^p} \mathbf{R}_{ij} \cdot \mathbf{R}_j^{ip}}{\sqrt{\sum_{v_j \in \mathcal{I}_i^p} \mathbf{R}_{ij}^2} \sqrt{\sum_{v_j \in \mathcal{I}_i^p} (\mathbf{R}_j^{ip})^2}}, \quad (5)$$

Similarly, we can compute the similarity s_i^n between u_i and his/her foe circle FO_i, and the similarity s_i^r between u_i and his/her random circle RA_i. Let $\mathbf{s}^p$, $\mathbf{s}^n$, and $\mathbf{s}^r$ be the similarity vectors of s_i^p, s_i^n, and s_i^r over users for these three circles, respectively. The means of $\mathbf{s}^p$, $\mathbf{s}^n$, and $\mathbf{s}^r$ are demonstrated in Table 2. We observe that (i) friend circles have larger means than foe circles; and (ii) among these three circles, friends circles have the largest means. From these two observations, we form two assumptions about social signed networks - (i) users are likely to be similar with their friend circles; and (ii) users are likely to be more similar with their friend circles than their foe circles.

For two vectors $\{\mathbf{x}, \mathbf{y}\}$, the null hypothesis H_0 and the alternative hypothesis H_1 of a two-sample t-test are defined as follows:

$$H_0 : \mathbf{x} <= \mathbf{y} \qquad H_1 : \mathbf{x} > \mathbf{y}. \quad (6)$$

where the null hypothesis indicates that the mean of $\mathbf{x}$ is less than or equal to that of $\mathbf{y}$. We perform t-test on $\{\mathbf{s}^p, \mathbf{s}^r\}$ and $\{\mathbf{s}^p, \mathbf{s}^n\}$ to examine aforementioned assumptions, respectively. For example, when we perform the t-test on $\{\mathbf{s}^p, \mathbf{s}^n\}$, the null hypothesis is that users are likely to be less similar with their friend circles than their foe circles; therefore if we reject the null hypothesis, then the assumption of users likely to be more similar with their friend circles than their foe circles is verified. The null hypothesis for each test is rejected at significance level $\alpha = 0.01$ with p-values shown in Table 3. The evidence from t-test on $\{\mathbf{s}^p, \mathbf{s}^r\}$ suggests that users are likely to be similar with their friend circles; and the evidence from t-test on $\{\mathbf{s}^p, \mathbf{s}^n\}$ indicates that users are likely to be more similar with their friend circles than their foe circles.

Table 3: P-values of t-test results.

Epinions			
	CI	COSINE	CI-COSINE
$\{\mathbf{s}^p, \mathbf{s}^r\}$	3.93e-124	6.07e-193	-2.71-111
$\{\mathbf{s}^p, \mathbf{s}^n\}$	3.12e-37	6.83e-65	2.35e-47
Slashdot			
	CI	COSINE	CI-COSINE
$\{\mathbf{s}^p, \mathbf{s}^r\}$	6.79e-140	5.62e-107	8.61e-85
$\{\mathbf{s}^p, \mathbf{s}^n\}$	1.83e-31	7.37e-27	3.89e-21

4. THE PROPOSED FRAMEWORK

Two types of information from unsigned social networks can be exploited for recommendation, which correspond to local information and global information [34]. Local information reveals the correlations among the user and his/her friends, while global information reveals the reputation of the user in the whole social network. Users in the physical world are likely to ask for suggestions from their local friends while they also tend to seek suggestions from users with high global reputation. This suggests that both local and global information can be exploited in social networks to improve the performance of recommender systems [31]. In the following subsections, we will first provide details about the methods for capturing local and global information in signed social networks, and then introduce the proposed RecSSN framework.

Matrix factorization is chosen as our basic model because it is one of the most popular techniques for building recommender systems [11, 10]. Assume that $\mathbf{U}_i \in \mathbb{R}^K$ is the K-dimensional preference latent factor of u_i, and $\mathbf{V}_j \in \mathbb{R}^K$ is the K-dimensional characteristic latent factor of item j. Typically, scores from u_i to v_j in $\mathbf{R}_{ij}$ are modeled by the interactions between their latent factors. This interaction is defined in terms of the product of the latent vectors:

$$\mathbf{R}_{ij} = \mathbf{U}_i^\top \mathbf{V}_j \quad (7)$$

Matrix factorization-based recommender systems solve the following optimization problem:

$$\min \sum_{i=1}^{N} \sum_{j=1}^{m} \mathbf{W}_{ij} \|\mathbf{R}_{ij} - \mathbf{U}_i^\top \mathbf{V}_j\|_2^2 + \alpha(\|\mathbf{U}\|_F^2 + \|\mathbf{V}\|_F^2) \quad (8)$$

where $\mathbf{U} = \{\mathbf{U}_1, \mathbf{U}_2, \ldots, \mathbf{U}_n\}$ and $\mathbf{V} = \{\mathbf{V}_1, \mathbf{V}_2, \ldots, \mathbf{V}_m\}$. $\mathbf{W}_{ij}$ controls the contribution from $\mathbf{R}_{ij}$, and the term $\|\mathbf{U}\|_F^2 + \|\mathbf{V}\|_F^2$ is added to avoid overfitting.

4.1 Capturing Local Information from Signed Social Networks

The local information in signed social networks is about the preference relations between users, and their "friends" (or users with positive links) and "foes" (or users with negative links). Next, we introduce our approach to capture local information from signed social networks based on the findings of the previous section.

Let P_i and N_i be u_i's friend circle, including users who have positive links with u_i, and foe circle, including users who have negative links with u_i, respectively. Based on P_i and N_i, we can divide users into three groups as below:

- OP includes users who have only positive links as - $OP = \{u_i | P_i \neq \emptyset \cap N_i = \emptyset\}$;
- ON includes users who have only negative links as - $ON = \{u_i | P_i = \emptyset \cap N_i \neq \emptyset\}$;
- PN contains users who have both positive and negative links as - $PN = \{u_i | P_i \neq \emptyset \cup N_i \neq \emptyset\}$.

We define U_i^p and U_i^n as the average user preferences of u_i's friend circle and foe circle, respectively, as follows:

$$\mathbf{U}_i^p = \frac{\sum_{u_j \in \mathcal{P}_i} \mathbf{S}_{ij}\mathbf{U}_j}{\sum_{u_j \in \mathcal{P}_i} \mathbf{S}_{ij}}, \quad \mathbf{U}_i^n = \frac{\sum_{u_j \in \mathcal{N}_i} \mathbf{S}_{ij}\mathbf{U}_j}{\sum_{u_j \in \mathcal{N}_i} \mathbf{S}_{ij}} \tag{9}$$

where $\mathbf{S}_{ij}$ is the connection strength between u_i and u_j. Next, we will discuss how to capture local information for these groups separately:

- For a user u_i with only friend circle (or $u_i \in OP$), our previous finding suggests that u_i's preference is likely to be similar with her friend circle. Hence, we force u_i's preference close to P_i by minimizing the following term:

$$\min \ \|\mathbf{U}_i - \mathbf{U}_i^p\|_2^2. \tag{10}$$

- For a user u_i with only foe circle (or $u_i \in ON$), this user is likely to be untrustworthy and we should not consider this user for the purpose of recommendation [37]. Therefore, we ignore local information from these users with only foe circles, which are only a small portion of the users in real-world signed social networks. For example, in the two studied datasets, there are less than 5% of users with only foe circles.

- For a user u_i with both friend and foe circles, our previous finding suggests that the preference of u_i is likely to be closer to that of his/her friend circle than that of his/her foe circle. In other words, (1) if a user u_i sits closer to his/her friend circle P_i than her foe circle N_i, i.e., $\|\mathbf{U}_i - \mathbf{U}_i^p\|_2^2 - \|\mathbf{U}_i - \mathbf{U}_i^n\|_2^2 < 0$, we should not penalize this case; while (2) if a user u_i sits closer to his/her foe circle N_i than her friend circle P_i, i.e., $\|\mathbf{U}_i - \mathbf{U}_i^p\|_2^2 - \|\mathbf{U}_i - \mathbf{U}_i^n\|_2^2 > 0$, we should add a penalty to pull u_i closer to P_i than N_i. Therefore, we propose the following minimization term to force u_i's preference closer to P_i than N_i as:

$$\min \ \max(0, \|\mathbf{U}_i - \mathbf{U}_i^p\|_2^2 - \|\mathbf{U}_i - \mathbf{U}_i^n\|_2^2) \tag{11}$$

Next, we give details on the inner workings of Eq. (11). (1) When u_i sits closer to his/her friend circle P_i than his/her foe circle N_i, the minimizing term in Eq. (11) is 0 because $\|\mathbf{U}_i - \mathbf{U}_i^p\|_2^2 - \|\mathbf{U}_i - \mathbf{U}_i^n\|_2^2 < 0$ and we do not add any penalty; and (2) when u_i sits closer to her foe circle N_i than her friend circle P_i, the minimizing term in Eq. (11) is $\|\mathbf{U}_i - \mathbf{U}_i^p\|_2^2 - \|\mathbf{U}_i - \mathbf{U}_i^n\|$ because $\|\mathbf{U}_i - \mathbf{U}_i^p\|_2^2 - \|\mathbf{U}_i - \mathbf{U}_i^n\| > 0$ and Eq. (11) will pull u_i back to P_i from N_i.

We can develop a unified term to capture local information from these three groups in signed social networks with the following observations - (1) if we define $\mathbf{U}_i^n = \mathbf{U}_i$ for u_i in OP, the term for OP is equivalent to $\max(0, \|\mathbf{U}_i - \mathbf{U}_i^p\|_2^2 - \|\mathbf{U}_i - \mathbf{U}_i^n\|_2^2)$; and (2) if we define $\mathbf{U}_i^n = \mathbf{U}_i$ for u_i in ON, the term $\max(0, \|\mathbf{U}_i - \mathbf{U}_i^p\|_2^2 - \|\mathbf{U}_i - \mathbf{U}_i^n\|_2^2)$ is 0 for ON, which indicates that we ignore the impact of users from ON. Therefore by redefining U_i^p and U_i^n as,

$$\mathbf{U}_i^p = \begin{cases} \frac{\sum_{u_j \in \mathcal{P}_i} \mathbf{S}_{ij}\mathbf{U}_j}{\sum_{u_j \in \mathcal{P}_i} \mathbf{S}_{ij}} & \text{for } u_i \in OP \cup PN, \\ \mathbf{U}_i & \text{for } u_i \in ON. \end{cases}$$
$$\mathbf{U}_i^n = \begin{cases} \frac{\sum_{u_j \in \mathcal{N}_i} \mathbf{S}_{ij}\mathbf{U}_j}{\sum_{u_j \in \mathcal{N}_i} \mathbf{S}_{ij}} & \text{for } u_i \in ON \cup PN, \\ \mathbf{U}_i & \text{for } u_i \in OP, \end{cases} \tag{12}$$

we can find a unified term to capture local information from signed social networks as:

$$\min \ \sum_{i=1}^{n} \max(0, \|\mathbf{U}_i - \mathbf{U}_i^p\|_2^2 - \|\mathbf{U}_i - \mathbf{U}_i^n\|_2^2) \tag{13}$$

4.2 Capturing Global Information from Signed Social Networks

The global information of a signed social network reveals the reputation of a user in the whole network [20]. User reputation is a sort of status that gives additional powers and capabilities in recommender systems [31]. There are many algorithms to calculate the reputations of nodes in positive networks [24, 8]. However, a small number of negative links can significantly affect the status of the nodes, which suggests that we should consider negative links. Therefore, we choose a variant of Pagerank, Exponential Ranking [36], taking into account negative links to calculate user reputations. In detail, we first perform Exponential Ranking to rank users by exploiting the global information of signed social networks. We assume that $r_i \in \{1, 2, \ldots, N\}$ is the reputation ranking of u_i where $r_i = 1$ denotes that u_i has the highest reputation in the social network. Then we define user reputation score $\mathbf{w}_i$ as a function f of user reputation ranking r_i: $\mathbf{w}_i = f(r_i)$ where the function f limits the value of the reputation score w_i within $[0, 1]$ and is a decreasing function of r_i, i.e., top-ranked users have high reputation scores.

In the physical world, user reputation plays an important role in recommendation. Many companies employ people with high reputations to enhance consumers' awareness and understanding of their products. Seno and Lukas found that suggestions from people with high reputations positively affect a consumer's adoption of a brand [26]. While in the online world, Massa found that recommendations from users with high reputations are more likely to be trustworthy [20]. To capture global information from signed social networks, we can use user reputation scores to weight the importance of their recommendations. Originally the importance of $\mathbf{R}_{ij}$ in Eq. (8) is controlled by $\mathbf{W}_{ij}$. With signed social networks, we should also consider the reputation of u_i; hence we define the new weight for $\mathbf{R}_{ij}$ as $\hat{\mathbf{W}}_{ij} = g(\mathbf{W}_{ij}, \mathbf{w}_i)$ where g is a function to combine two weights. With these new weights, the formulation to capture global information from signed social networks is computed as follows:

$$
\min \sum_{i=1}^{N} \sum_{j=1}^{m} g(\mathbf{W}_{ij}, \mathbf{w}_i) \|\mathbf{R}_{ij} - \mathbf{U}_i^\top \mathbf{V}_j\|_2^2 + \alpha(\|\mathbf{U}\|_F^2 + \|\mathbf{V}\|_F^2) \tag{14}
$$

where the importance of $\mathbf{R}_{ij}$ is controlled by $\mathbf{W}_{ij}$ and the reputation score of u_i through a function g.

5. AN OPTIMIZATION ALGORITHM FOR RECSSN

We have introduced our approaches to capture local and global information from signed social networks. With these model components, we propose a recommendation framework, RecSSN, which exploits local and global information simultaneously from signed social networks. The proposed RecSSN framework solves the following optimization problem:

$$
\min \sum_{i=1}^{N} \sum_{j=1}^{m} g(\mathbf{W}_{ij}, \mathbf{w}_i) \|(\mathbf{R}_{ij} - \mathbf{U}_i \mathbf{V}_j^\top)\|_2^2 + \alpha(\|\mathbf{U}\|_F^2 + \|\mathbf{V}\|_F^2) + \beta \sum_{i=1}^{n} \max(0, \|\mathbf{U}_i - \mathbf{U}_i^p\|_2^2 - \|\mathbf{U}_i - \mathbf{U}_i^n\|_2^2) \tag{15}
$$

where $\beta \sum_{i=1}^{n} \max(0, \|\mathbf{U}_i - \mathbf{U}_i^p\|_2^2 - \|\mathbf{U}_i - \mathbf{U}_i^n\|_2^2)$ captures local information from signed social networks and the parameter β controls its contribution. The term $g(\mathbf{W}_{ij}, \mathbf{w}_i)$ is introduced to capture global information from signed social networks.

By setting $g(\mathbf{W}_{ij}, \mathbf{w}_i) = \mathbf{W}_{ij}$ and ignoring all negative links, the proposed formulation for RecSSN in Eq. (15) can be written as follows:

$$
\min \sum_{i=1}^{N} \sum_{j=1}^{m} \mathbf{W}_{ij} \|(\mathbf{R}_{ij} - \mathbf{U}_i \mathbf{V}_j^\top)\|_2^2 + \alpha(\|\mathbf{U}\|_F^2 + \|\mathbf{V}\|_F^2) + \beta \sum_{i=1}^{n} \|\mathbf{U}_i - \mathbf{U}_i^p\|_2^2 \tag{16}
$$

Interestingly, this formulation is equivalent to one of the state-of-the-art recommender systems with positive networks SocialMF [6]. Therefore, RecSSN provides a unified recommendation framework with unsigned and signed social networks.

Eq. (15) is jointly convex with respect to $\mathbf{U}$ and $\mathbf{V}$ and there is no nice solution in closed form due to the use of the max function. A local minimum can be obtained through following gradient decent optimization method, which usually works well for recommender systems [11]. We define $\mathbf{M}_i^k$ at the k-th iteration for u_i as follows:

$$
\mathbf{M}_i^k = \begin{cases} 1 & \|\mathbf{U}_i - \mathbf{U}_i^p\|_2^2 - \|\mathbf{U}_i - \mathbf{U}_i^n\|_2^2 > 0 \\ 0 & \text{otherwise} \end{cases} . \tag{17}
$$

Then, we use J to denote the objective function of Eq. (15) in the k-th iteration as follows:

$$
\begin{aligned}
J = & \sum_{i=1}^{N} \sum_{j=1}^{m} g(\mathbf{W}_{ij}, \mathbf{w}_i) \|(\mathbf{R}_{ij} - \mathbf{U}_i \mathbf{V}_j^\top)\|_2^2 \\
& + \alpha(\sum_{i=1}^{N} \|\mathbf{U}\|_2^2 + \sum_{j=1}^{m} \|\mathbf{V}_j\|_2^2) + \beta \sum_{i=1}^{N} \mathbf{M}_i^k (\\
& \|\mathbf{U}_i - \frac{\sum_{u_j \in \mathcal{P}_i} \mathbf{S}_{ij} \mathbf{U}_j}{\sum_{u_j \in \mathcal{P}_i} \mathbf{S}_{ij}}\|_2^2 - \|\mathbf{U}_i - \frac{\sum_{u_j \in \mathcal{N}_i} \mathbf{S}_{ij} \mathbf{U}_j}{\sum_{u_j \in \mathcal{N}_i} \mathbf{S}_{ij}}\|_2^2)
\end{aligned} \tag{18}
$$

The derivatives of J with respect to $\mathbf{U}_i$ and $\mathbf{V}_j$ are as follows:

$$
\begin{aligned}
\frac{\partial J}{\partial \mathbf{U}_i} = & -2 \sum_{j} g(\mathbf{W}_{ij}, \mathbf{w}_i)(\mathbf{R}_{ij} - \mathbf{U}_i \mathbf{V}_j^\top) \mathbf{V}_j + 2\alpha \mathbf{U}_i \\
& + 2\beta \mathbf{M}_i^k (\mathbf{U}_i - \mathbf{U}_i^p) - 2\beta \mathbf{M}_i^k (\mathbf{U}_i - \mathbf{U}_i^n) \\
& - 2\beta \sum_{u_j \in \mathcal{P}_i} \mathbf{M}_j^k (\mathbf{U}_j - \mathbf{U}_j^p) \frac{1}{\sum_{u_j \in \mathcal{P}_i} \mathbf{S}_{ji}} \\
& + 2\beta \sum_{u_j \in \mathcal{N}_i} \mathbf{M}_j^k (\mathbf{U}_j - \mathbf{U}_j^n) \frac{1}{\sum_{u_j \in \mathcal{N}_i} \mathbf{S}_{ji}} \\
\frac{\partial J}{\partial \mathbf{V}_j} = & -2 \sum_{i} g(\mathbf{W}_{ij}, \mathbf{w}_i)(\mathbf{R}_{ij} - \mathbf{U}_i \mathbf{V}_j^\top) \mathbf{U}_i + 2\alpha \mathbf{V}_j
\end{aligned} \tag{19}
$$

The detailed algorithm is shown in Algorithm 1. In Algorithm 1, γ_u and γ_v are learning steps, which are chosen to satisfy Goldstein Conditions [23]. Next, we briefly discuss the algorithm. In line 1, we initialize latent factors of users $\mathbf{U}$ and items $\mathbf{V}$ randomly. In each iteration, we calculate $\mathbf{U}_i^p$, $\mathbf{U}_i^n$ and $\mathbf{M}_i^k$ for u_i from line 3 to line 6. From line 7 to line 9, we update $\mathbf{U}$ and $\mathbf{V}$ using aforementioned update rules. After learning the user preference matrix $\mathbf{U}$ and the item characteristic matrix $\mathbf{V}$ via Algorithm 1, an unknown score $\hat{\mathbf{R}}_{i'j'}$ from the user u_i' to the item v_j' will be predicted as $\hat{\mathbf{R}}_{i'j'} = \mathbf{u}_{i'}^\top \mathbf{v}_{j'}$.

Algorithm 1: The Proposed Recommendation Framework RecSSN with Signed Social Networks.

Input: The rating information $\mathbf{R}$, positive links $\mathbf{A}_n$, negative links $\mathbf{A}_p$, the number of latent factors K and β
Output: The user preference matrix $\mathbf{U}$ and the item characteristic matrix $\mathbf{V}$

```
1:  Initialize U and V randomly and set k = 1
2:  while Not convergent do
3:     for i = 1 : N do
4:        Calculate U_i^p and U_i^n according to Eq. (12)
5:        Calculate M_i^k according to Eq. (17)
6:     end for
7:     Calculate ∂J/∂U and ∂J/∂V
8:     Update U ← U − γ_u ∂J/∂U
9:     Update V ← V − γ_v ∂J/∂V
10:    k = k + 1
11: end while
```

6. EXPERIMENTAL RESULTS

In this section, we conduct experiments to answer the following two questions - (1) can the proposed RecSSN framework improve the recommendation performance by exploiting signed social networks? and (2) which model components of RecSSN contribute to the performance improvement? Before answering these questions, we begin by introducing the experimental settings.

6.1 Experimental Settings

In Epinions, the scores in the user-item matrix denote the rating scores from users to items. Following common ways to assess recommendation performance in rating systems, we choose two metrics, corresponding to the Root Mean Square Error (RMSE) and the Mean Absolute Error (MAE), which are formally defined as follows:

$$RMSE = \sqrt{\frac{\sum_{(u_i,v_j)\in\mathcal{T}}(\mathbf{R}_{ij} - \hat{\mathbf{R}}_{ij})^2}{|\mathcal{T}|}},$$
$$MAE = \frac{1}{|\mathcal{T}|}\sum_{(u_i,v_j)\in\mathcal{T}}|\mathbf{R}_{ij} - \hat{\mathbf{R}}_{ij}|, \tag{20}$$

where $\mathcal{T}$ is the set of ratings in the testing set, $|\mathcal{T}|$ is the size of $\mathcal{T}$ and $\hat{\mathbf{R}}_{ij}$ is the predicted rating from u_i to v_j. A smaller RMSE or MAE value means better performance. Note that previous work demonstrated that *small improvement in RMSE or MAE terms can have a significant impact on the quality of the top few recommendations* [9]. In this work, we choose x% of rating scores as training and the remaining $1 - x$% as testing, and x is varied as $\{50, 70, 90\}$.

In Slashdot, scores in the user-item matrix indicate whether users are associated with certain items. In this scenario, the performance is often evaluated via precision@N and recall@N [27], which are formally defined as follows:

$$precision@N = \frac{\sum_{u_i\in\mathcal{U}}|TopN_i \bigcap I_i|}{\sum_{u_i\in\mathcal{U}}|TopN_i|} \tag{21}$$
$$recall@N = \frac{\sum_{u_i\in\mathcal{U}}|TopN_i \bigcap I_i|}{\sum_{u_i\in\mathcal{U}}|I_i|}, \tag{22}$$

where $TopN_i$ is the set of N items recommended to user u_i that u_i has not been associated in the training set, and I_i is the set of items that have been associated with u_i in the testing set. A larger precision@N or recall@N value means better performance. The values of precision@N and recall@N are usually small in the case of sparse datasets. For example, the precision@5 is less than 0.05 over a dataset with $8.02e-3$ density [40]. In this work, we set $N = 5$ and $N = 10$.

6.2 Performance Comparison of Recommender Systems

To answer the first question, we compare the proposed RecSSN framework with existing recommender systems. Traditional collaborative filtering systems can be grouped into memory-based systems and model-based systems; hence we choose two groups of baseline methods.

The first group of baseline methods includes the following memory-based systems:

- **UCF**: This system makes recommendations by aggregating recommendations from ones' similar users only based on the user-item matrix.
- **pUCF**: This system is a variant of **UCF**, which combines recommendations from ones' similar users and their friends [20]. **pUCF** utilizes both user-item matrix and positive links.
- **pnUCF**: This system is a variant of **pUCF**, which excludes recommendations from ones' foes by exploiting negative links [37]. **pnUCF** makes use of user-item matrix, positive and negative links.

The second group of baseline methods includes the following model-based systems:

- **MF**: This system performs matrix factorization on the user-item matrix as shown in Eq. (8) [25]. It only utilizes the user-item matrix.
- **SocialMF**: This system combines both user-item matrix and positive links for recommendation [6], which is a special case of the proposed framework with only positive links as shown in Eq. (16).
- **SoReg**: This system also leverages both user-item matrix and positive links, and defines social regularization to capture positive links [17].
- **LOCABAL**: This system captures local and global information of positive links under the matrix factorization framework [31].
- **disSoReg**: In [15], two systems are proposed to exploit positive and negative links, respectively. **disSoReg** is a combination of these two systems to exploit positive and negative links simultaneously, which is actually a variant of **SoReg** by considering negative links as dissimilarity measurements.

Note that we use cross-validation to determine parameters for all baseline methods. For RecSSN, β is set to 0.7 and 0.3 for Epinions and Slashdot, respectively. More details about parameter selection for RecSSN will be discussed in the following subsections. We empirically set $\alpha = 0.1$ and the number of latent factors $K = 10$ for both datasets. In Eq. (14), we empirically find that $f(x) = \frac{1}{log(x+1)}$ and $g(x, y) = x - y$ work well. The comparison results are demonstrated in Tables 4 and 5 for Epinions and Slashdot, respectively.

We make the following observations:

- In general, model-based methods outperform memory-based methods on the two studied datasets. Most of the existing recommender systems suffer from the data sparsity problem but model-based methods are usually less sensitive than memory-based methods [9].
- **pUCF** outperforms **UCF**. Furthermore, **SocialMF**, **SoReg** and **LOCABAL** outperform **MF**. These results support the known contention that exploiting positive links can significantly improve recommendation performance.
- **LOCABAL** exploits local and global information from positive links, and obtains better performance than the systems which model only local information from positive links such as **SocialMF** and **SoReg**. These observations indicate the importance of global information for recommendation.

Table 4: Comparison of Different Recommender Systems in Epinions

Training	Metrics	Memory-based Methods			Model-based Methods					
		UCF	**pUCF**	**pnUCF**	**MF**	**SocialMF**	**SoReg**	**LOCABAL**	**disSoReg**	RecSSN
50%	MAE	1.0323	0.9764	0.9683	1.0243	0.9592	0.9589	0.9437	0.9679	0.9273
	RMSE	1.2005	1.1477	1.1392	1.1902	1.1397	1.1354	1.1212	1.1407	1.0886
70%	MAE	1.0074	0.9493	0.9402	0.9988	0.9341	0.9327	0.9274	0.9425	0.8981
	RMSE	1.1758	1.1301	1.1196	1.1692	1.1163	1.1127	1.1009	1.1237	1.0697
90%	MAE	0.9817	0.9272	0.9187	0.9779	0.9189	0.9153	0.9017	0.9263	0.8863
	RMSE	1.1592	1.1059	1.0885	1.1525	1.0986	1.0951	1.0821	1.1032	1.0479

Table 5: Comparison of Different Recommender Systems in Slashdot

Metrics	Memory-based Methods			Model-based Methods					
	UCF	**pUCF**	**pnUCF**	**MF**	**SocialMF**	**SoReg**	**LOCABAL**	**disSoReg**	RecSSN
P@5	0.0343	0.0372	0.0381	0.0354	0.0387	0.0386	0.0394	0.0379	0.0419
R@5	0.0438	0.0479	0.0485	0.0453	0.0492	0.0488	0.0498	0.0473	0.0511
P@10	0.0332	0.0358	0.0364	0.0338	0.0365	0.0368	0.0375	0.0359	0.0388
R@10	0.0413	0.0454	0.0463	0.0427	0.0463	0.0467	0.0479	0.0457	0.0497

pnUCF obtains better performance than **pUCF**, which suggests that excluding recommendations from users with negative links can improve recommendation performance. Furthermore, **disSoReg** performs worse than **SoReg**. These results suggest that we may not consider negative links as dissimilarities in recommendation, which is consistent with observations in [33].

The proposed RecSSN framework always obtains the best performance. RecSSN captures local and global information from signed social networks. In addition to positive links, signed social networks also provide negative links. More details about the effects of negative links on the performance of RecSSN will be discussed in the following subsection.

With these observations, we can draw conclusions about the first question - the proposed RecSSN framework outperforms the state-of-the-art recommender systems by exploiting local and global information from signed social networks.

6.3 Impact of Negative Links on RecSSN

We will now focus on the second issue of examining the precise impact of negative links on RecSSN. The experimental results in the previous subsection show that the proposed RecSSN framework outperforms various representative recommender systems with unsigned social networks. Compared to these systems, RecSSN also leverages information from negative links. In this subsection, we investigate the impact of negative links on the proposed RecSSN framework to answer the second question. In particular, we eliminate the effects of negative links systematically from RecSSN by defining the following algorithmic variants:

RecSSNnGN - Eliminating the effect of negative links from global information of signed social networks by using Pagerank to calculate status scores of users with only positive links.

RecSSNnLN - Eliminating the effect of negative links from local information of signed social networks by replacing $\sum_{i=1}^{n} \max(0, \|\mathbf{U}_i - \mathbf{U}_i^p\|_2^2 - \|\mathbf{U}_i - \mathbf{U}_i^n\|_2^2)$ with $\sum_{i=1}^{n} \|\mathbf{U}_i - \mathbf{U}_i^p\|_2^2$ in Eq. (15).

RecSSNnGN-LN - Eliminating the effects of negative links from global and local information of signed social networks.

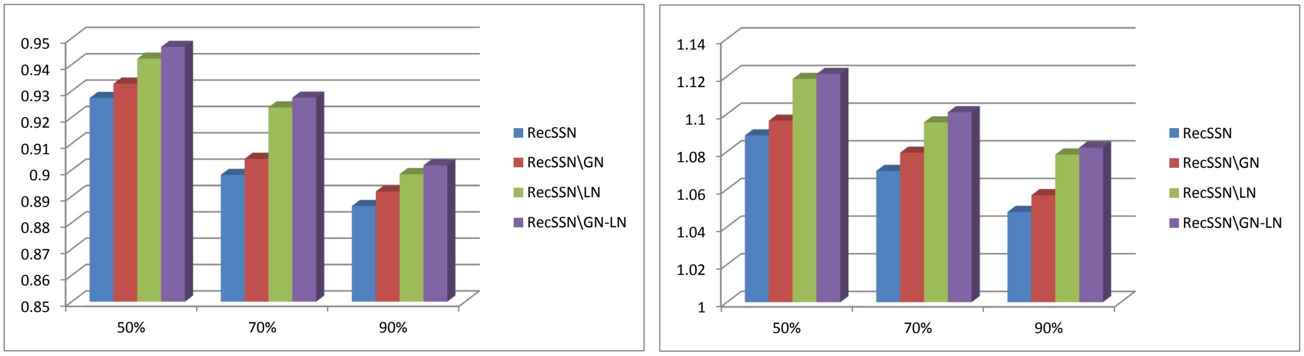
The parameters in all these variants are determined via cross-validation. The experimental results in Epinions are demonstrated in Figure 1. Note that we only show the results in Epinions because similar results were obtained in Slashdot. In general, eliminating any model component which captures the effect of negative links will reduce the recommendation performance. The relative performance reductions for variants compared to RecSSN are shown in Table 6. When eliminating the effect of global information of negative links from the proposed framework, the performance of *RecSSNnGN* degrades. We make a similar observation for *RecSSNnLN* when eliminating the effect of local information. For example, compared to RecSSN, *RecSSNnGN* and *RecSSNnLN* have 1.02% and 3.06% relative performance reductions, respectively, in terms of RMSE with 50% of Epinions data. When eliminating the effects of negative links from global and local information of signed social networks, *RecSSNnGN-LN* obtains worse performance than both *RecSSNnGN* and *RecSSNnLN*. This suggests that local and global information contain complementary information to each other for recommendation.

With the results from Figure 1 and Table 6, we can answer the second question - both local and global information of negative links in the proposed RecSSN framework can help improve the recommendation performance.

6.4 Parameter Analysis for RecSSN

The parameter β controls the contribution of local information in signed social networks. In this subsection, we investigate how changes of β affect the performance of RecSSN. We vary the value of β as $\{0, 0.001, 0.01, 0.1, 0.3, 0.5, 0.7, 1, 10\}$. The results in Epinions w.r.t. RMSE and MAE are demonstrated in Figures 2(a) and 2(b), respectively. Since we have similar observations in Slashdot, we only show the results in Epinions to save space.

With increase in β, the importance of local information is increased. We make the following observations:



(a) AME (b) RMSE

Figure 1: Impact of Negative Links on The Proposed Framework RecSSN in Epinions.

Table 6: Relative Performance Reductions for Variants Compared to RecSSN.

Variants	50%		70%		90%	
	MAE	RMSE	MAE	RMSE	MAE	RMSE
RecSSNnGN	-0.88%	-1.02%	-0.98%	-1.21%	-0.92%	-1.15%
RecSSNnLN	-2.06%	-3.06%	-3.15%	-2.71%	-1.67%	-3.21%
RecSSNnGN-LN	-2.59%	-3.29%	-3.56%	-3.22%	-2.04%	-3.56%

The performance rst increases rapidly, which suggests that local information is helpful in improving recommendation performance in signed social networks.

When β varies from 0.3 to 0.7, the performance is relatively stable. This property is useful from a practical point of view because it makes it easier to set β.

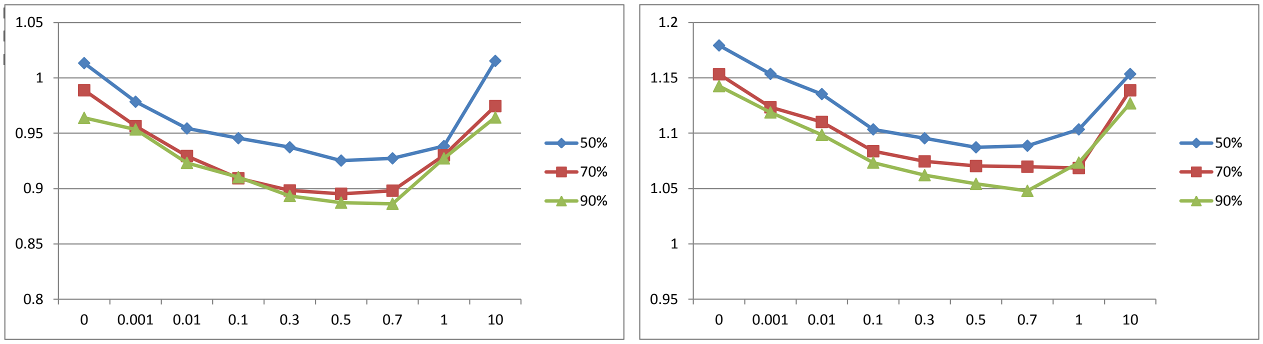
After this point, the performance reduces. When β increases from 1 to 10, the performance reduces dramatically. A large value of β will lead to local information dominating the learning process. In such cases, the estimates of the user preference matrix $\mathbf{U}$ and the item characteristic matrix $\mathbf{V}$ will over t to the local information in signed social networks. For example, when β ! 1, the user preference matrix $\mathbf{U}$ is learned only from signed social networks and the item characteristic matrix $\mathbf{V} = 0$.

7. RELATED WORK

The pervasive nature of social media provides independent sources of information, which brings new opportunities for recommendation. Recently, social relations have found increasing importance from the perspective of improving recommendation performance [20, 16, 17, 7]. In [16], a matrix-factorization system, referred to as SoRec, is proposed. It performs a co-factorization on the user-item ratings matrix and user-user social relation matrix by assuming that users should share the same user preference vectors in the rating space and the social relation space. Trust Ensemble is introduced in [18] to take advantage of strong dependency connections. It assumes that a user's online behavior can be a ected by his/her trusted friends on the Web, and, based on this intuition, unknown ratings for a certain user are predicted by the user's characteristics and the user's trusted friends' recommendation. In [6], a social recommender system with trust propagation is proposed to recommend items for users in social network. The underlying assumption of this method is that directly connected users may have similar interests and thus it forces a user's preference close to the average user preference of his/her social network. Social regularization is employed by [17] to exploit strong dependency connections for recommendation. This approach forces a user's preference close to user preferences of his/her social networks. The low cost of social relation formation can lead to social relations with heterogeneous strengths [38]. Since users with strong strength are more likely to share similar tastes than those with weak strength, treating all social relations equally is likely to lead to degradation in recommendation performance. Therefore the closeness between a user's preference and the preferences of his/her social network is controlled by their rating similarities [17]. These social recommender systems can reduce the number of cold-start users and improve recommendation performance [6].

8. CONCLUSIONS

The pervasively available social networks in social media have encouraged a large body of literature about recommendation. The vast majority of these recommender systems focus on unsigned social networks (or social networks with only positive links). However, social networks in social media could contain positive and negative links and little work exists for recommendation with signed social networks. The leveraging of negative links for recommendation is a challenging task because straightforward extensions of unsigned networks do not seem to be applicable in this case. In this paper, we rst perform data-driven analysis on signed social networks and make a number of observations. Then we provide principled approaches to capture local and global information from signed social networks mathematically, which results in a novel recommendation framework, which we refer to as RecSSN. Experimental results demonstrate that the proposed framework outperforms various state-of-the-



(a) MAE (b) RMSE

Figure 2: Performance Variations of The Proposed RecSSN Framework w.r.t. β in Epinions.

art recommender systems. Further experiments are conducted to understand the importance of signed social networks in the proposed RecSSN framework.

There are several directions, which might be investigated. First, the proposed RecSSN framework chooses matrix factorization as the basic model on top of which the algorithms are constructed. While this is a natural choice because of the well-known robustness of such systems, it would be instructive to investigate whether other types of models can be used. Second, as user preferences and signed social networks might evolve, incorporating temporal information into the proposed RecSSN framework is an interesting direction. Third, we make several important observations about signed social networks in this paper, which may be helpful in developing algorithms for other online applications of signed social networks, such as information propagation and spammer detection. Finally a comprehensive overview about signed network mining in [30] suggests that mining signed networks is still in its early stage; thus we would like to investigate more applications in signed networks.

Acknowledgments

This material is based upon work supported by, or in part by, the U.S. Army Research O ce (ARO) under contract/grant number 025071, the O ce of Naval Research (ONR) under grant number N000141010091, and the Army Research Laboratory and was accomplished under Cooperative Agreement Number W911NF-09-2-0053. The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the o cial policies, either expressed or implied, of the Army Research Laboratory or the U.S. Government. The U.S. Government is authorized to reproduce and distribute reprints for Government purposes notwithstanding any copyright notation here on.

9. REFERENCES

[1] K.-Y. Chiang, N. Natarajan, A. Tewari, and I. S. Dhillon. Exploiting longer cycles for link prediction in signed networks. In *Proceedings of the 20th ACM international conference on Information and knowledge management*, pages 1157{1162. ACM, 2011.

[2] J. Cho. The mechanism of trust and distrust formation and their relational outcomes. *Journal of Retailing*, 82(1):25{35, 2006.

[3] J. Golbeck. Generating predictive movie recommendations from trust in social networks. *Trust Management*, pages 93{104, 2006.

[4] R. Guha, R. Kumar, P. Raghavan, and A. Tomkins. Propagation of trust and distrust. In *Proceedings of the 13th international conference on World Wide Web*, pages 403{412. ACM, 2004.

[5] C.-J. Hsieh, K.-Y. Chiang, and I. S. Dhillon. Low rank modeling of signed networks. In *Proceedings of the 18th ACM SIGKDD international conference on Knowledge discovery and data mining*, pages 507{515. ACM, 2012.

[6] M. Jamali and M. Ester. Trustwalker: a random walk model for combining trust-based and item-based recommendation. In *Proceedings of the 15th ACM SIGKDD international conference on Knowledge discovery and data mining*, pages 397{406. ACM, 2009.

[7] M. Jiang, P. Cui, R. Liu, Q. Yang, F. Wang, W. Zhu, and S. Yang. Social contextual recommendation. In *CIKM*. ACM, 2012.

[8] J. M. Kleinberg. Authoritative sources in a hyperlinked environment. *Journal of the ACM (JACM)*, 46(5):604{632, 1999.

[9] Y. Koren. Factorization meets the neighborhood: a multifaceted collaborative ltering model. In *Proceeding of the 14th ACM SIGKDD international conference on Knowledge discovery and data mining*, pages 426{434. ACM, 2008.

[10] Y. Koren. Collaborative ltering with temporal dynamics. *Communications of the ACM*, 53(4):89{97, 2010.

[11] Y. Koren, R. Bell, and C. Volinsky. Matrix factorization techniques for recommender systems. *Computer*, 42(8):30{37, 2009.

[12] J. Kunegis, J. Preusse, and F. Schwagereit. What is the added value of negative links in online social networks? In *Proceedings of the 22nd international conference on World Wide Web*, pages 727{736. International World Wide Web Conferences Steering Committee, 2013.

[13] J. Kunegis, S. Schmidt, A. Lommatzsch, J. Lerner, E. W. De Luca, and S. Albayrak. Spectral analysis of signed graphs for clustering, prediction and visualization. In *SDM*, volume 10, pages 559{559. SIAM, 2010.

[14] J. Leskovec, D. Huttenlocher, and J. Kleinberg. Predicting positive and negative links in online social networks. In *Proceedings of the 19th international conference on World wide web*, pages 641{650. ACM, 2010.

[15] H. Ma, M. R. Lyu, and I. King. Learning to recommend with trust and distrust relationships. In *Proceedings of the third ACM conference on Recommender systems*, pages 189{196. ACM, 2009.

[16] H. Ma, H. Yang, M. Lyu, and I. King. Sorec: social recommendation using probabilistic matrix factorization. In *Proceeding of the 17th ACM conference on Information and knowledge management*, pages 931{940. ACM, 2008.

[17] H. Ma, D. Zhou, C. Liu, M. Lyu, and I. King. Recommender systems with social regularization. In *Proceedings of the fourth ACM international conference on Web search and data mining*, pages 287{296. ACM, 2011.

[18] N. Ma, E. Lim, V. Nguyen, A. Sun, and H. Liu. Trust relationship prediction using online product review data. In *Proceeding of the 1st ACM international workshop on Complex networks meet information & knowledge management*, pages 47{54. ACM, 2009.

[19] P. Marsden and N. Friedkin. Network studies of social in uence. *Sociological Methods and Research*, 22(1):127{151, 1993.

[20] P. Massa. A survey of trust use and modeling in real online systems. *Trust in E-services: Technologies, Practices and Challenges*, 2007.

[21] D. H. McKnight and N. L. Chervany. Trust and distrust de nitions: One bite at a time. In *Trust in Cyber-societies*, pages 27{54. Springer, 2001.

[22] M. McPherson, L. Smith-Lovin, and J. Cook. Birds of a feather: Homophily in social networks. *Annual review of sociology*, pages 415{444, 2001.

[23] J. Nocedal and S. Wright. *Numerical optimization*. Springer verlag, 1999.

[24] L. Page, S. Brin, R. Motwani, and T. Winograd. The pagerank citation ranking: Bringing order to the web. 1999.

[25] R. Salakhutdinov and A. Mnih. Probabilistic matrix factorization. *Advances in neural information processing systems*, 20:1257{1264, 2008.

[26] D. Seno and B. Lukas. The equity e ect of product endorsement by celebrities: A conceptual framework from a co-branding perspective. *European Journal of Marketing*, 2007.

[27] B. Sigurbjörnsson and R. Van Zwol. Flickr tag recommendation based on collective knowledge. In *Proceedings of the 17th international conference on World Wide Web*, pages 327{336. ACM, 2008.

[28] M. Szell, R. Lambiotte, and S. Thurner. Multirelational organization of large-scale social networks in an online world. *Proceedings of the National Academy of Sciences*, 107(31):13636{13641, 2010.

[29] J. Tang, S. Chang, C. Aggarwal, and H. Liu. Negative link prediction in social media. In *ACM International Conference on Web Search and Data Mining*, 2015.

[30] J. Tang, Y. Chang, C. Aggarwal, and H. Liu. A survey of signed network mining in social media. *arXiv preprint arXiv:1511.07569*, 2015.

[31] J. Tang, X. Hu, H. Gao, and H. Liu. Exploiting local and global social context for recommendation. In *Proceedings of the Twenty-Third international joint conference on Arti cial Intelligence*, pages 2712{2718. AAAI Press, 2013.

[32] J. Tang, X. Hu, and H. Liu. Social recommendation: a review. *Social Network Analysis and Mining*, 3(4):1113{1133, 2013.

[33] J. Tang, X. Hu, and H. Liu. Is distrust the negation of trust? the value of distrust in social media. In *ACM Hypertext conference*, 2014.

[34] J. Tang and H. Liu. Trust in social computing. In *Proceedings of the companion publication of the 23rd international conference on World wide web companion*, pages 207{208. International World Wide Web Conferences Steering Committee, 2014.

[35] J. Tang, J. Tang, and H. Liu. Recommendation in social media: recent advances and new frontiers. In *Proceedings of the 20th ACM SIGKDD international conference on Knowledge discovery and data mining*, pages 1977{1977. ACM, 2014.

[36] V. Traag, Y. Nesterov, and P. Van Dooren. Exponential ranking: Taking into account negative links. *Social Informatics*, pages 192{202, 2010.

[37] P. Victor, C. Cornelis, M. De Cock, and A. Teredesai. Trust-and distrust-based recommendations for controversial reviews. In *Web Science Conference (WebSci'09: Society On-Line)*, number 161, 2009.

[38] R. Xiang, J. Neville, and M. Rogati. Modeling relationship strength in online social networks. In *Proceedings of the 19th international conference on World wide web*, 2010.

[39] B. Yang, W. K. Cheung, and J. Liu. Community mining from signed social networks. *Knowledge and Data Engineering, IEEE Transactions on*, 19(10):1333{1348, 2007.

[40] M. Ye, X. Liu, and W. Lee. Exploring social in uence for recommendation-a probabilistic generative approach. In *Annual International ACM SIGIR Conference on Research and Development in Information Retrieval*, pages 325{334, 2012.