How Placing Limitations on the Size of Personal Networks Changes the Structural Properties of Complex Networks

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ABSTRACT
People-to-people interactions in the real world and in virtual environments (e.g., Facebook) can be represented through complex networks. Changes of the structural properties of these complex networks are caused through a variety of dynamic processes. While accepting the fact that variability in individual patterns of behavior (i.e., establishment of random or FOAF-type potential links) in social environments might lead to an increase or decrease in the structural properties of a complex network, in this paper, we focus on another factor that may contribute to such changes, namely the size of personal networks. Any personal network comes with the cost of maintaining individual connections. Despite the fact that technology has shrunk our world, there is also a limit to how many close friends one can keep and count on. It is a relatively small number. In this paper, we develop a multi-agent based model to capture, compare, and explain the structural changes within a growing social network (e.g., expanding the social relations beyond one's social circles). We aim to show that, in addition to various dynamic processes of human interactions, limitations on the size of personal networks can also lead to changes in the structural properties of networks (i.e., the average shortest-path length). Our simulation result shows that the famous small world theory of interconnectivity holds true or even can be shrunk, if people manage to utilize all their existing connections to reach other parties. In addition to this, it can clearly be observed that the network's average path length has a significantly smaller value, if the size of personal networks is set to larger values in our network growth model. Therefore, limitations on the size of personal networks in network growth models lead to an increase in the network's average path length.

Categories and Subject Descriptors
I.6.6 [**Computing Methodologies**]: Simulation Output Analysis

Keywords
Small-world network; Complex Networks; Average path length; Size of personal networks; Network growth model

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1. INTRODUCTION
Social network theory views social relationships in terms of nodes and ties. Nodes are the individuals within the networks, and ties are the relationships between them. As we know from literature on social network analysis, the degree of a node in a network is a count of the number of unique edges that are connected to the node. Although the meaning of this expression is extremely simple, there are challenges associated with measuring the node degree. For example, if someone asks you to write down the names of people you actually know, you can write down a long list for sure. However, by changing the nature of the question slightly to something like write down the names of people you actually trust (rather than you know), your reactions would be different. Instead of writing down names immediately, you ask about the context in which trust should be considered. The reason is that trust is a context specific issue. Talking about people you know is about revealing your social circle in the society and the people you meet in your daily life. However, talking about people you trust is about revealing your personal preferences in different contexts. When we talk about someone that we trust, we think about an attractiveness measure for him or her with respect to different contexts such as love, family, cooking, and sport. The number of people we know is higher than those we trust, because we believe that more personal relationships provide us with more opportunity but an attractiveness threshold for each relationship filters them and, therefore, reduce the number of connected people.

The person-to-person interactions between all of the individuals in our society form a large social network of people. These days, people-to-people interactions have been extended through social media and virtual communities such as Facebook and Twitter. Hence, we can argue that the technology is shrinking our world. The famous small world theory of personal relationships comprises the idea of being connected to any other person by a chain of only five people in average (i.e., six degree of separation). Recent studies have even shown that the six degrees of separation has shrunken, due to social networking tools (e.g., Facebook, Twitter) [1].

Assuming a social network is a product of its people's interactions, the first question that comes to mind is what kinds of interactions take place within it and how these interactions can be categorized. Interactions among people of a network can be regarded as dynamic processes, which lead to changes in its structural properties. Dynamic processes in turn can be categorized into two groups: (1) the process, which occurs during the growth of a network and represents the tendency of new users to establish links to other members upon entry into the network; (2) the process, which occur among existing users of a network in order to establish links between them. The process, which occurs among users of a network when establishing potential links, is further divided into two categories: (a) the first category includes the establishment of potential links of a Friend-Of-A-Friend type (FOAF type) with social distance equal to 1; and (b) the second category includes the establishment of random potential links with other users, who are present in the network. While accepting the fact that variability in individual patterns of behavior in social environments lead to a change in the structural properties of a network, we focus on another factor that may contribute to such changes in this paper. We focus on the size of personal networks.

A personal network in the real world comes with the cost of maintaining the relationships. Therefore, the question rises what could be a genuine size of a personal network? Is it the so-called Dunbar number (i.e., the rule of 150 connections) [2]? If we think about the size of personal networks, we can agree on that the sizes are different among people. The agreement for variation is based on the fact that individuals have various abilities to make friendship connections. Furthermore, we naturally let go of old ones, which no longer work, at different rates. Therefore, the fraction of the friends that we contact regularly and the fraction of the friends that we find sufficiently attractive for long-term relationships are different across people. We can say that our close friends are people that have a mutual attraction with us. No matter what kinds of attractions causes a true relationship, as it may vary from person to person, the most important fact is that (1) it could be explored and developed over time and that (2) the attractiveness threshold is different from person to person. The attraction concept gives people a chance of link establishments. For example, if we want to have a great deal of control over the ones who truly matter, we need to serve a purpose to them. Otherwise, they will terminate the relationship. In this regard, the maintenance of our personal network is not costless. It requires spending time and effort to maintain such a relationship. This way, we gradually figure out who our real and close friends are. Consequently, there is a limit on how many close friends one can have.

Besides, social interactions among people form social networks of friends. In the real world, our first-degree connections are the people that we personally know and our second-degree connections are friends of our friends (FOAF). In the real world, it is unrealistic to assume that we know the third-degree connections (i.e., the friends of our FOAF). Social networking platforms such as Facebook, however, provide us with the opportunity of navigating larger chains of connections.

If we consider our trusted contacts only, the can reach less people only due to their low number. In this context, the small world theory has been criticized. One of the most popular criticisms is related to the type of the item that was sent to the target people during Milgram's experiments [3, 4]. In fact, Kleinfeld argues that what the type of item (e.g., passport or letter) could make a significant difference in whether and how it reaches their targets [5]. Although the focus of Kleinfeld's discussion is on the incentive to forward the item, we think that, depending on the type of the item, it can even be propagated through different kinds of people. For example, with respect to a regular letter, the letter could be transferred through people we know, but, with respect to a passport, the passport would be transferred through trusted contacts only. As we discussed, the number of trusted people is a relatively small number and, therefore, requires more steps. Having said that, existing research on "small world" theory focused only on calculating the average shortest path length of networks that utilize all the existing connections among people [1, 6]. Therefore, placing limitations on the size of personal networks distinguishes our analysis of the network's average path length from previous analyses in literature.

Taking each of the discussed key elements into account, we propose the following features that a network growth model should incorporate:

1. Variability in individual patterns of behavior: The establishment of links of different types needs to be considered due to the difference of behavior of people in social environments.
2. Different rate of variability of new node entrance and link establishments: The rate, at which people join a network, is different than the rate of link creation among existing ones.
3. Limitations on the size of personal networks: The number of trusted contacts is smaller than the number of known contacts.

With this, we follow the idea that human behavior is the key to formulate a realistic network growth model. To test these realistic network growth models and to measure the structural property (i.e., the average path length) of the network as a function of time, we developed a simulation environment.

This simulation environment is also used to answer our research question. We ask how a limitation in the number of trusted contacts impacts the structural properties of the entire network. In detail, we investigate the extent, to which a limitation on the size of personal networks (i.e., having a few trusted friends only) leads to an increase in the average shortest path length of the entire network. The simulation results clearly show that the size of the personal network plays a definite role in the formation of a network. The average shortest-path length, which provides a measure of how close the individuals within the network are, increases as the size of the personal network decreases.

One of the main essential contributions of our work is that we can explain the differences in the average shortest path length measured in empirical studies and existing network growth models. Our model is able to capture, compare, and explain the structural changes within a growing social network with respect to social characteristics of individuals in more detail.

The remainder of this paper is organized as follows. In section 2, we discuss related research and theoretical background on the topic. In section 3, we detail the model and its parameters. Experimental results are presented in section 4. Finally, we present our conclusion and discussion in section 5.

2. THEORETICAL BACKGROUND

With the help of technology, we have managed to shrink the world. It serves us with finding more potential opportunities (e.g., finding a friend). In other words, it reduces the distance between two people or, in technical terms, it makes the average shortest-path length among the individuals smaller. The emerging network, which is a product of its people's interactions, can be seen as a map that connects each of us with other people. Since 1991, it has been an important issue to find out to what extent people are connected.

Stanley Milgram, in his famous experiments [3, 4], was interested in computing the distance distribution of the acquaintance graph. The main conclusions outlined in Milgram's paper were that, depending on the sample of people chosen, the average path length of individuals within the network is smaller than expected. Despite the existence of some empirical studies on the small world theory, the results obtained in various empirical environments are not consistent with the magic number six.

Dodds et al. performed a global social search experiment to replicate small-world hypothesis and showed that social searches could reach their targets in a median of five to seven steps. They classified different types of relationships and observed their frequencies and strength. The result of their analysis showed that senders preferred to take advantage of friendship rather than family or business ties. It also indicated the fact that the origin of relationships mainly appears to be family, work, or school affiliations. Furthermore, strengths of the relationships were fairly close. Therefore, we can say that the most useful category of social ties were medium-strength friendships that originated in social environments [7]. Backstrom et al. repeated Milgram's experiment by using the entire Facebook network and reported the observed average distance of 4.74, corresponding to 3.74 intermediaries or "degree of separation" [1]. The study indicates the fact that various externalities such as geography have the potential to change the degree of locality among the individuals and, finally, increase or decrease the average path length. Ugander et al. studied the anatomy of the social graph of Facebook and computed several features of that [6]. Their main observations showed that the degrees of separation between any two Facebook users are smaller than the commonly cited six degrees, and, even more, it has been shrinking over time. They also found that the graph neighborhood of users has a dense structure. Furthermore, they found that there is a modular community structure driven by nationality.

The result reported by Backstrom et al. can be considered as admissible evidence that technology can shrink the world. However, we argue that just because the network among people becomes denser than before, it does not mean that there is an increase in the trust between the individuals within the network. People still have to manage their connections. It requires spending time and effort to maintain them. Another issue, which should be considered, is that, if we do not have many people that we can really trust among our existing connections, we are not able to send critical information. Moreover, if we want to forward critical information from a start point to a target point in another part of the network, we cannot utilize all the existing connections because we need to spend our time and effort productively. Actually that is a point where Kleinfeld argues that the low success rate in Milgram experiments is disappointing [5]. Some experiments revealed a low rate of chain completion and majority of chains died before reaching the target point. He considered the possibility that people could have gotten connected but they just did not bother to forward the information to other intermediaries. Therefore, in this paper, we build our own hypothesis based on the discussion so far. We test the hypothesis that putting limitation on the size of personal networks (having few trusted friends) lead to an increase or decrease in the structural properties of a complex network.

Among the literature reviews related to the size of personal network, we can point to the work performed by Aristotle, who noted that warm friendship is only possible with a few people [8]. Therefore, "The number of one's close friends must be limited". Several studies pointed to the fact that the maintenance of social networks is not costless and, depending on the type of the network being modeled, it results in cut-offs in real networks [9, 10, 11, 12].

Our hypothesis in this research contributes and plays a major role to the existing research in the sense that we stress the fact that having limitation on size of personal network leads to changes in the structural properties of a complex network. With respect to our contribution, a generative model [17] is proposed for analyzing the influence of having limitation on the size of personal network on the complex network's baseline properties. This paper engages with the idea that human behavior is the key to formulate a network growth model. It is reasonable to assume that variability in individual patterns of behavior in social environments is the base for a model of network growth. Having said that, it is also reasonable to accept the fact that maintenance of our personal network is not costless and it requires spending time and effort. However, the extent, to which this limitation affects the network's structural properties, is still unclear in literature on complex networks.

3. MODEL

3.1 Simulation Parameters

The experimental settings for our model are as follows: We conduct a multi-agent-based simulation in Netlogo [13], in order to answer our research question. The proposed model is a generative model based on the ideas that individual pattern of behavior in social environment is the key. This model can generate a network, in which the members follow the classical preferential attachment (i.e., attractiveness of each individuals is modeled by preferential attachment rule [15, 16]) for connecting to other users. Furthermore, the users in this model have the ability to create potential links. Preferential attachment rule is used to model a situation, where some people have more attractions compared to others. A preferential attachment rule says that a new vertex is linked with already existing ones with probabilities proportional to their degrees. For this purpose, three parameters (i.e., P_{GM}, P_{FOAF} and P_{RAN}) are used, which represent the rate of a network growth, the rate of establishing potential links of FOAF-type, and the rate of establishing potential links of random type, respectively. The value of P_{GM} determines the rate of a network growth. For example, if $P_{GM} = 0.25$, it signifies that the rate of newly entered individuals is 25% and for 75% individuals have the chance of creating links among themselves.

The value of P_{RAN} is assumed to be equal to 1 - P_{FOAF}. Thus, if the value of the P_{FOAF} parameter equals 1, no random link formation process exists in the generative model. These parameters have values in the range [0, 1] and represent the rate of establishing potential links of random and FOAF-type. For example, if $P_{FOAF} = 0.5$, it signifies that the probability of random link formation or conversion of a link with social distance of 2 to a link with social distance of 1 is 50%. The size of the personal network (S_{PN}) is set to 5, 10, 15, and 20, respectively. Such a network modeling approach enables us to simulate and profoundly comprehend the dynamic transformations of a network and its effects on the network's structural properties.

3.2 Structural Properties of Networks

In this paper, we analyze one of the main structural properties of a network, called average shortest-path length (AVL). The shortest-path length (AVL) is defined as the shortest distance between node pairs in a network [14]. Therefore, the average shortest-path length, AVL, is defined as shown in the following equation:

$$AVL = \frac{1}{\frac{1}{2}N(N-1)} \sum_{i \geq j} l_{ij} \tag{1}$$

where N is the number of nodes, and l_{ij} is the shortest-path length between node i and j.

3.3 Simulation Environment

The graphical user interface (GUI) of our simulation environment is depicted in Figure 1. The GUI of our simulation consists of a two-dimensional field that contains nodes and their connections.

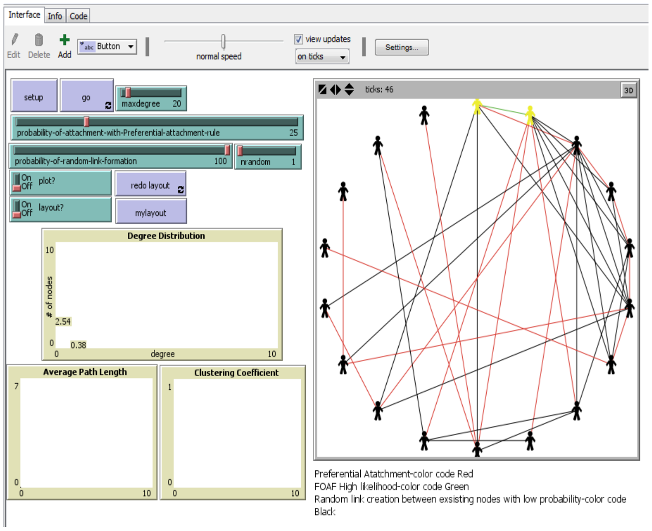

Figure 1. GUI of the proposed model developed in NetLogo.

4. SIMULATION RESULTS

The properties of the networks derived from our network growth model are computed over 10 suits of experiments and the average results are plotted in the diagrams shown. Figure 2 depicts that the AVL value increases with an increase in P_{GM} value (i.e., rate of a network growth with preferential attachment rule). Since, in real scenarios, the rate at which people join a network is much shorter than the rate of link creation among the existing ones, it is safe to assume that most of the existing dynamic processes within a social network are related to the establishment of potential links of FOAF or random type. It means that the rate of newly entered individuals is much lower than the rate of creation of links among the existing individuals. Therefore, we consider the value of P_{GM} to be lower than P_{FOAF} variable. The value of P_{GM} is set to 0.25, while the value of P_{FOAF} is selected from the range [0.25, 0.75]. Figure 2 shows the obtained AVL of the networks derived from our generative model with P_{GM} = 0.25 and a P_{FOAF} value within the range [0, 1].

In order to see the effect of limitations on the size of personal network, we set the S_{PN} value to 5, 10, 15, and 20, respectively, and repeated the experiments. As mentioned earlier, the P_{RAN} value is always assumed to be the complement of P_{FOAF}. The model was tested with forty configurations for each P_{FOAF} value, which were compared with one another. The result is presented in Figure 3(A-C). The x-axis shows the simulation period, while the y-axis represents the AVL value. In the early stage of the simulation, the network's average path length has a significantly greater variability in its value than later. As time goes by, the fluctuations of the curves are little.

Considering Figure 3(A-C), it is clear that the AVL values consistently decline with increase in value of S_{PN}. It means that the size of personal networks must be large enough, in order to have smaller AVL value among the population. Therefore, applying more limitation on the size of personal networks leads to an increase in AVL value. The obtained results indicate the fact that, if the size of personal networks are relatively small (S_{PN}=5), the AVL value among the population tends to be large. It also shows that network's average path length has a significantly smaller variability in its value, if the size of personal networks is set to larger values (S_{PN}=10, 15, 20). Therefore, as the series of figures show, in addition to a different rate of variability of link establishments, a limitation on the size of personal networks also leads to changes in the structural properties of networks (i.e., the average shortest-path length). The simultaneous impact of the rate of variability in potential link establishments (P_{FOAF}) and the limitation on the size of personal networks (S_{PN}) suggests that the AVL value reaches its minimum with an increase in S_{PN} and decrease in P_{FOAF}. Such a result in its own turn is evidence for the importance of potential links of random type for the formation of a network with smaller average path lengths value.

Compared to Figure 2, Figure 3(A-C) exhibits a trend. It is noteworthy that the results of AVL values are different. The ranges of AVL values of Figure 2 are between 2.9 and 3.2. The ranges of values of Figure 3(A-C) are in the range [2.9, 4.6], [3.14, 5.06], and [3.3, 5.6], respectively. The discrepancy in the results reflects the influence of size of personal networks on the whole network's structural properties.

5. DISCUSSION AND CONCLUSION

With the advent of social networking platforms, we witnessed the emergence of a new paradigm in friendship patterns among people. No matter where we are in the world, social networking platforms are able to shrink our world. They make the world a smaller place by bringing people together. They form an important part of online activities and networking. Although we agree upon the fact that our world is getting smaller and smaller, there is also a limit to how many close friends one can keep and count on. Besides, having a high number of friends does not necessarily increase our trust circles. In order to keep the boundary for our relationships, we usually determine features and patterns that distinguish a friend and a trusted contact. In addition to this, any personal network in the real world comes with the cost of maintaining the individual connections. Maintenance of our personal network is not costless. It requires spending time and effort. For example, the low success rate in Milgram experiments and the reported average length of eight in his "communication project" might be due to a natural limit in the number of trusted contacts, which the sampled people have had.

Following this idea, we can state that it is an unrealistic assumption to think that people can utilize all their existing connections to reach other people. By looking deeply at the nature of the research questions in small world theory experiments, one can easily understand that in case of forwarding critical information to the target person, the sender cannot utilize all of his existing connections.

In order to reduce the risk of failure in sending critical information, the source and all the intermediaries have to utilize their trusted contacts, which is a relatively small number. Therefore, we test the hypothesis that putting limitation on the size of personal networks (having few trusted friends) leads to a change in the structural properties (i.e., the average shortest path) of the entire network of people.

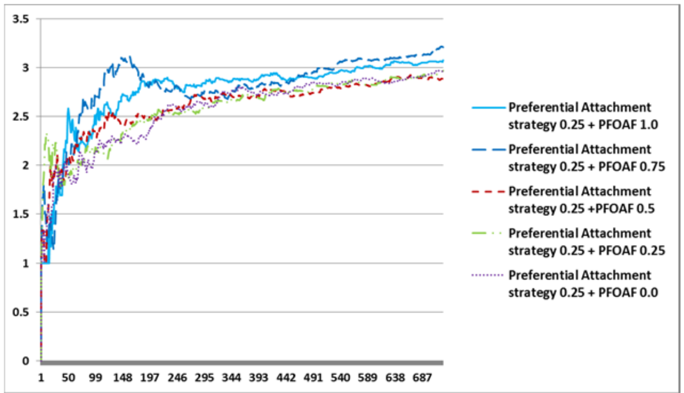

Figure 2. Changes in AVL with respect to the preferential attachment growth model are shown. The x-axis depicts the simulation period, while the y-axis represents the AVL value.

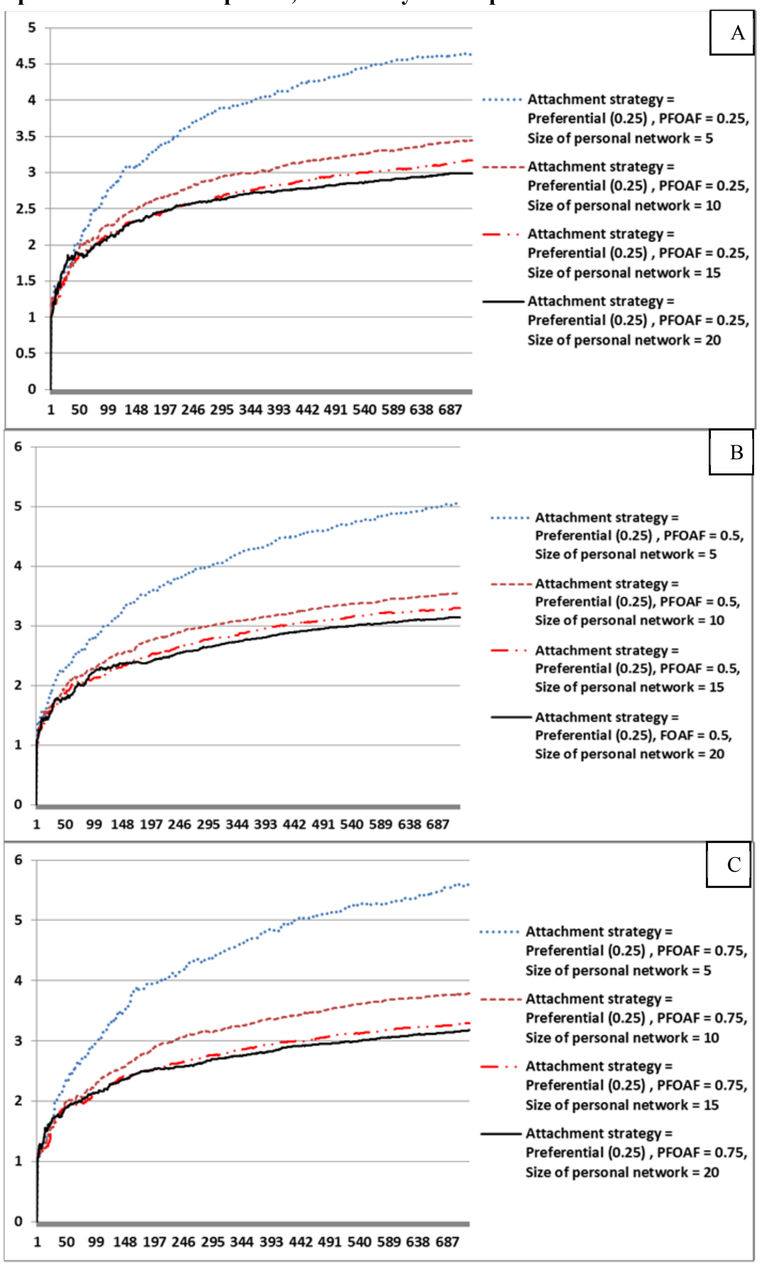

Figure 3 (A-C). Changes in AVL with respect to the preferential attachment growth model for different sizes of the personal networks. The x-axis shows the simulation period, while the y-axis represents the AVL

For answering the research question, we present a new network growth model that considers the size of the personal network. Our model is able to capture and compare the structural changes within a growing social network with respect to certain social characteristics of individuals. Since social phenomena are complex, we followed an agent-based modeling approach to depict a complex structure emerging from the interaction of many simple parts over time.

We conducted numerical simulations to calculate and compare the average shortest-path length of individuals within the generated network. The model was tested with forty configurations, which were compared with each other. It was observed that, in addition to a different rate of variability in individual patterns of behavior in social environments, the limitations on the size of personal networks significantly changes the average path lengths among individuals. As clearly observed, the network's average path length had a significantly smaller value, if the size of the personal networks has been set to a large value. Therefore, limitations on the size of personal networks lead to an increase in AVL value.

The main essential implication of our research results is that we can explain the differences in the average shortest path length measured in empirical studies and existing network growth models (which did not consider the size of the personal network before). Our model is able to explain the structural changes within a growing social network with respect to social characteristics of individuals in more detail.

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