Rumor Spreading and Inoculation of Nodes in Complex Networks

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ABSTRACT

Over the Internet or on social networks rumors can spread and can affect the society in disaster. The question one asks about this phenomenon is that whether these rumors can be suppressed using suitable mechanisms. One of the possible solutions is to inoculate a certain fraction of nodes against rumors. The inoculation can be done randomly or in targeted fashion. In this paper, small world network model has been used to investigate the efficiency of inoculation. It has been found that if average degree of small world network is small than both inoculation methods are successful. When average degree is large, neither of these methods are able to stop rumor spreading. But if acceptability of rumor is reduced along with inoculation, the rumor spreading can be stopped even in this case.The proposed hypothesis has been verified using simulation experiments.

Categories and Subject Descriptors

J.4 [**Computer Applications**]: Social and Behavioral Sciences—*Sociology*

General Terms

Theory

Keywords

Rumor spreading model, Complex networks, Random and targeted inoculation

1. INTRODUCTION

When a disaster occurred, people try to acquire information through social web. Sometimes, social web information is useful, so people can get valuable knowledge. On the other hand, social web information is occasionally unreliable. Harmful rumors spread and cause people to panic. In today's information oriented society, a mechanism to suppress harmful rumors in social web has become very important.

In last decade many complex network models e.g. ER model, small world network model, scale free network model have been introduced [1, 8]. Work has also been done on epidemic spreading in complex networks [4, 5, 7]. Some kind of networks, where most nodes have similar degrees, i.e., the degree distribution has small fluctuations e.g. in random networks, regular networks and small world networks, are called homogeneous networks. In contrary to homogeneous networks, the networks with large fluctuations in degree distributions are called heterogeneous networks, e.g. scale free networks [14].

The rumor spreading can be seen as epidemic spreading. Work on epidemic spreading has been done by many researchers [5, 7]. In these epidemic spreading, epidemic inoculation is attractive. Pastor-Satorras and Vespignani [9] found that random inoculation was not efficient for all complex networks like in scale free networks. Therefore, targeted inoculation scheme was found to be successful inoculation for scale free networks. An standard model of rumor spreading, was introduced by Daley and Kendal [2] and its variants such as Maki-Thomsan [6]. In Daley-Kendal(DK) population is subdivided into three groups: ignorant, spreaders and stiflers. The rumor is propagated through the population by pairwise contacts between spreaders and other individuals in the population. Any spreader involved in a pairwise meeting attempts to infect other individual with the rumor. In Maki Thomsan (MK) model when spreader contacts another spreader only initiating spreader becomes a stifler. DK and MK model have an important shortcoming is that it do not take into account the topology of the underlying social interconnection networks along which rumors spread. To consider the topology of network, rumor model on small world network [6, 11, 12] and scale free networks [3] have been defined. Using mean field theory Nekovee, *et al.* [6] discovered that threshold (below which a rumor can not be spread) was small in homogeneous networks. Few studies have been done to stop rumor spreading. These studies are more important since false and fatal rumors have negative impacts on the society in disaster.

This paper proposes a framework to suppress the harmful rumors over social web by using small world network model. Watts Strogatz(WS) model for small world network [10] integrates the features of regular and random graphs together. They can be created by starting with some regular graphs (e.g. a lattice ring) and randomly "rewiring" a given fraction, P, of edges as described by Watts and Strogatz.

2. PROPOSED RUMOR SPREADING MODEL

Nekovee, *et al.* [6] gave a general stochastic model for the rumor spreading. In this model, the total population is divided into three compartments: ignorant individuals, spreaders and stiflers. Ignorant populations are susceptible to being informed, spreaders spread the rumor and stiflers

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know the rumor but they are not interested in spreading it. In this paper, stiflers have been further divided into two compartments: one population of stiflers who accept the rumor but not interested in spreading it, other ones related to population who reject the rumor (not interested in accepting), they can be considered as inoculated population as in epidemic spreading model. There are N nodes and each node can be in one of the compartments of ignorant, spreaders and stiflers. When a spreader meets with an ignorant node, ignorant node becomes a spreader with rate λ,or a stifler who accepts the rumor with rate α, or a stifler who rejects the rumor with rate δ. When a spreader meets with a spreader or stifler, the spreader becomes a stifler who accepts the rumor with rate γ. The quality of this model is that it allows ignorant nodes to become stifler when it is contacted by a spreader.This is similar to real life where a person may not spread rumor when he or she hears it from an other person. Here $1/\delta$ can be considered as acceptability of rumor. If we increase δ, ignorant nodes are more likely to become stiflers who do not accept the rumor. If for a rumor δ is small, the rumor has more acceptable information. In this paper $I(t), S(t), R_{acc}(t), R_{rej}(t)$ represent the fraction of ignorant nodes, spreaders, stiflers who accept the rumor and stiflers who rejects the rumor respectively, as the function of time t. $R(t)$ is the total number of stiflers (who accept and reject rumor) at time t. Above rumor spreading process can be summarized by following set of pairwise interaction.

$$\begin{aligned}
&I(t) + S(t) + R_{acc}(t) + R_{rej}(t) = 1, \qquad (1)\\
&I + S \xrightarrow{\lambda} 2S,\\
&I + S \xrightarrow{\delta} R_{rej} + S,\\
&I + S \xrightarrow{\alpha} R_{acc} + S,\\
&S + R_{acc} \xrightarrow{\gamma} 2R_{acc},\\
&S + R_{rej} \xrightarrow{\gamma} R_{acc} + R_{rej},\\
&S + S \xrightarrow{\gamma} R_{acc} + S.
\end{aligned}$$

Following the mean field rate equations given in [6], we define mean field rate equations for our model as

$$\begin{aligned}
\frac{dI(t)}{dt} &= -(\lambda+\alpha+\delta)\langle k\rangle I(t)S(t),\\
\frac{dS(t)}{dt} &= \lambda\langle k\rangle I(t)S(t) - \gamma\langle k\rangle S(t)(S(t)+R_{acc}(t)+\\
&\quad R_{rej}(t)),\\
\frac{dR_{acc}(t)}{dt} &= \gamma\langle k\rangle S(t)(S(t)+R_{acc}(t)+R_{rej}(t))+\\
&\quad \alpha\langle k\rangle I(t)S(t),\\
\frac{dR_{rej}(t)}{dt} &= \delta\langle k\rangle I(t)S(t). \qquad (2)\\
R(t) &= R_{acc}(t)+R_{rej}(t). \qquad (3)
\end{aligned}$$

where, $\langle k\rangle$ is the average degree of the WS network and initial conditions of above equations are $I(0) \approx 1, S(0) \approx 0, R_{acc}(0) = 0, R_{rej}(0) = 0$. In equation (2), $\lambda + \alpha + \delta \leq 1$.

By using equations (1), (2) and (3), one get the following transcendental equation (Appendix A),

$$R(\infty) = 1 - e^{-\frac{\lambda+\gamma}{\gamma}R(\infty)} \qquad (4)$$

where, $R(\infty) = \lim_{t\to\infty} R(t)$ and $R(\infty) = R_{acc}(\infty)+R_{rej}(\infty)$ One can solve the Equation(1) with simulink and get a relation between fraction of different population with the time as shown in Figure (1). We know that for a nonzero solu-

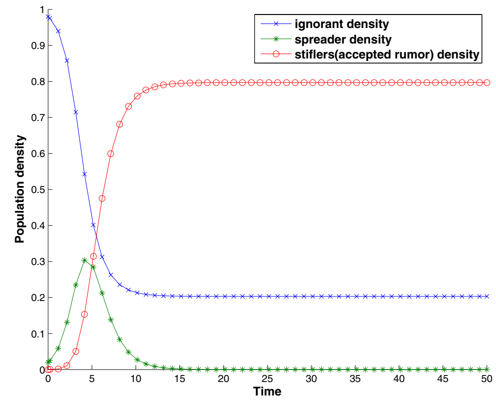

Figure 1: Ignorant, Spreaders and R_{acc} densities with time.

tion of equation (4), $(\lambda+\gamma)/\gamma \geq 1$. This inequality is always valid, except for γ=0. There will not be any threshold for λ. It is different from SIR [4, 13] model. Now we can also solve expressions for $R_{acc}(\infty)$ and $R_{rej}(\infty)$:

$$\begin{aligned}
R_{acc}(\infty) &= \frac{\alpha+\lambda}{\delta+\alpha+\lambda}R(\infty)\\
R_{rej}(\infty) &= \frac{\delta}{\delta+\alpha+\lambda}R(\infty) \qquad (5)
\end{aligned}$$

Using equation (5):

$$\frac{R_{acc}(\infty)}{R_{rej}(\infty)} = \frac{\alpha+\lambda}{\delta} \qquad (6)$$

From equation (6) it is evident that if we increase δ (decrease the acceptability) and fix other parameters, $R_{acc}(\infty)$ will decrease. Thus, if we decrease the acceptability of the rumor, density of populations who accept the rumor will also decrease.

3. RANDOM INOCULATION ON SMALL WORLD NETWORK

A Random inoculation strategy inoculates a fraction of the nodes randomly, using no knowledge of the network. g defines the fraction of inoculative nodes. In mean field level, in case of uniform inoculation, the initial conditions of equation (2) will be modified as: $I(0) \approx 1-g, S(0) \approx 0, R_{acc}(0) = 0, R_{rej}(0) = g$. Solving the equations (1) , (2) and (3) under these initial conditions, following transcendental equation (Appendix A) is obtained.

$$R(\infty) = 1 - (1-g)e^{\frac{\lambda+\gamma}{\gamma}g}e^{-\frac{\lambda+\gamma}{\gamma}R(\infty)} \qquad (7)$$

In equation (7), g always has a nonzero solution (Appendix B). Defining equation (7) as an auxiliary function we get:

$$f(R(\infty)) = 1 - (1-g)e^{\frac{\lambda+\gamma}{\gamma}g}e^{-\frac{\lambda+\gamma}{\gamma}R(\infty)} - R(\infty)$$

$$f'(R(\infty)) = \frac{\lambda+\gamma}{\gamma}(1-g)e^{\frac{\lambda+\gamma}{\gamma}g}e^{-\frac{\lambda+\gamma}{\gamma}R(\infty)} - 1$$

As g is one of the nonzero solution of equation (7)

$$f'(g) = \frac{\lambda + \gamma}{\gamma}(1 - g) - 1 \leq 0$$

Therefore, we will be able to get the critical inoculation value of g, $g_c = \frac{\lambda}{\lambda+\gamma}$. When, $g > g_c$, $R(\infty) = g$, is only the nonzero solution of equation (7). Therefore, $R_{acc}(\infty) = R(\infty) - R_{rej}(\infty) = R(\infty) - g \equiv 0$. This model shows that, by using random inoculation the density of stiflers who accept the rumor can be brought down to zero.

The above analysis can be validated by numerical simulations on WS model. Let N=10,000, $\langle k \rangle = 4$ and rewiring probability (P)=0.8. Set the other parameters as, $\lambda = 0.25, \alpha = 0, \delta = 0$ and $\gamma = 0.25$. These simulations are performed for 100 different initial configurations of proposed rumor models on at least 10 different realizations of WS model. Results are shown in Figure (2), it can be analyzed that if

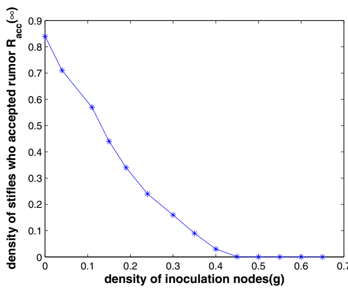

Figure 2: Comparison between R_{acc} and density of inoculation nodes g in random inoculation with small degree 4.

density of inoculation g increases, the density of stiflers who accept the rumor decreases monotonically. Critical inoculation g_c is approx 0.5 in Figure (2), which is in the agreement of the calculated value of $g_c = \frac{\lambda}{\lambda+\gamma} = 0.5$ considered.

By given this analysis it can be seen that critical inoculation g_c does not depends on $\langle k \rangle = 4$ of homogeneous networks. If our mean field equations can correctly describe in proposed rumor model, it should satisfy $0 \leq (\lambda + \alpha + \delta)\langle k \rangle S(t) \leq 1$ and $0 \leq \gamma \langle k \rangle (S(t) + R_{acc}(t) + R_{rej}(t)) \leq 1$ [2], when $\langle k \rangle$ is too large, these constraints will fail. Therefore, solution to calculate g_c needs to be re investigated. We will again perform numerical simulation on WS model with parameters as previous and varying the value of δ, $0 \leq \delta \leq 0.75$. The results are plotted in Figure (3). It has been observed from Figure (3) that when $\lambda = 0.25, \alpha = 0, \delta = 0, \gamma = 0.25$, analysis to calculate g_c will fail. The obtained g_c is approx 0.85 which is much higher than the previous obtained value 0.5 for $\delta = 0$. If mean field rate equations are valid, then constraint $0 \leq \gamma \langle k \rangle (S(t) + R_{acc}(t) + R_{rej}(t)) \leq 1$ should be satisfied, when $\gamma = 0.25$, $\langle k \rangle = 12$ and $S(0) + R_{acc}(0) + R_{rej}(0) = g_c = 0.5$ but $\gamma \langle k \rangle (S(0) + R_{acc}(0) + R_{rej}(0)) = 1.5 > 1$. Therefore mean field rate equation will not be accurate when $\langle k \rangle$ is large and random inoculation is no longer efficient. The problem can be solved by increasing the parameter δ (decrease the acceptability of rumor) and apply random inoculation method at same time. In Figure (3), when $\delta = 0.75$ and g=0.55, $R_{acc}(\infty)$ near to zero, which means that the given model can be effective by increasing the value of δ.

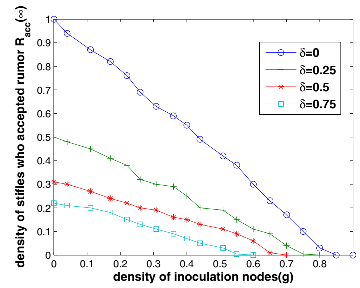

Figure 3: Comparison between R_{acc} and density of inoculation nodes g in random inoculation with large degree 12.

4. TARGETED INOCULATION ON SMALL WORLD NETWORK

To spread of epidemics on heterogeneous networks like scale free, targeted inoculation were introduced. If we have information about all degrees, we may rank nodes by degree, and use targeted inoculation to inoculate nodes by descending degree. When a high-degree node is inoculated, the effective degree of its neighbors drops. This inoculation strategy is more effective on heterogeneous networks e.g. scale free networks. For numerical simulation for targeted inoculation,in WS model by using same parameters we will get results in Figure (4). It can be analyzed for the Figure (4)

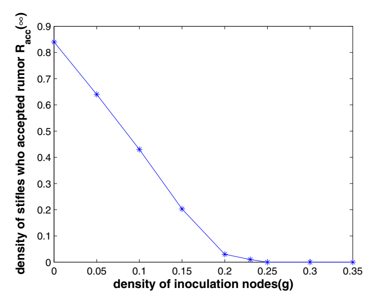

Figure 4: Comparison between R_{acc} and density of inoculation nodes g in targeted inoculation with small degree 4.

that the target inoculation is better than the random inoculation. Here g_c is approx 0.25 and rumor spreading is almost zero. The degree distribution in WS model is Poisson degree distribution, which is not strictly homogeneous and possesses some heterogeneous property is there. If a small world network has strictly homogeneous than random inoculation will be equivalent to targeted inoculation.

When $\langle k \rangle$ is high again, simulate the result on WS model with $\langle k \rangle = 12$ fix all parameters as previous and vary δ . In Figure (5) we can visualize that when $\langle k \rangle$ is large, targeted inoculation is not effective. By increasing the parameter δ with targeted inoculation, rumor spreading can be suppressed.

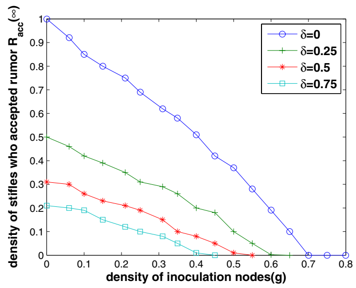

Figure 5: Comparison between R_{acc} and density of inoculation nodes g in targeted inoculation with large degree 12.

5. CONCLUSION

In this paper a new compartment of nodes as stiflers who rejects the rumor with rate δ is added . In real it is possible that one ignorant node with the meeting of spreader node can directly loose the interest to spread the rumor with or without accepting it. It have been shown that proposed model supports the small critical inoculation value g_c in random and targeted inoculation to control rumor spreading when $\langle k\rangle$ of small world network is small. It is also found in targeted inoculation that g_c is smaller than random inoculation when $\langle k\rangle$ is small, even when the distribution conductivities over small world networks is relatively uniform. When $\langle k\rangle$ is very high, mean field approximation fails. In this case alone random or targeted inoculation will not be effective. Therefore, one should decrease the acceptability of the rumor and apply on the random or targeted inoculation method simultaneously. After doing this we got a small value of g_c to control rumor spreading, which was very high in the case of high acceptability ($\delta = 0$). Hence, in the presented method there is no need to inoculate large number of nodes.

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APPENDIX

A. PROOF OF EQUATION (4) AND (7)

By using initial conditions of our model:
From Equation (2):

$$\begin{aligned} \int_0^t \frac{1}{I(\tau)}\,dI(\tau) &= -(\lambda+\alpha+\delta)\langle k\rangle \int_0^t S(\tau)\,d\tau, \\ I(t) &= I(0)e^{[-(\lambda+\alpha+\delta)\langle k\rangle \int_0^t S(\tau)\,d\tau]}. \end{aligned} \tag{8}$$

From Equation (1) and (3):

$$\frac{dR(t)}{dt} = \gamma\langle k\rangle S(t)[1-I(t)] + (\alpha+\delta)\langle k\rangle I(t)S(t)] \tag{9}$$

From Equation (9) :

$$\begin{aligned} \frac{dR(t)}{dt} &= \gamma\langle k\rangle S(t)(S(t)+R(t)) + \\ & \quad \langle k\rangle I(t)S(t)(\alpha+\delta) \end{aligned} \tag{10}$$

From Equation (1),(3) and (10) and integrating:

$$\langle k\rangle \int_0^t S(\tau)\,d\tau = \frac{R(\infty)}{\gamma}\left\{\frac{\lambda+\gamma}{\lambda+\alpha+\delta}\right\} \tag{11}$$

Put value of Equation (11) into Equation (8):

$$R(\infty) = 1 - e^{-\frac{\lambda+\gamma}{\gamma}R(\infty)}$$

Similarly by using initial conditions for g fraction of initial inoculation of nodes in our model:

$$R(\infty) = 1-(1-g)e^{\frac{\lambda+\gamma}{\gamma}g}e^{-\frac{\lambda+\gamma}{\gamma}R(\infty)}$$

B. EQUATION (7) HAS NON ZERO SOLUTION *g*

Let $\frac{\lambda+\gamma}{\gamma} = \beta$,

$$R(\infty) = 1-(1-g)e^{\beta(g-R(\infty))}$$

If $(g - R(\infty)) \geq 0$, $R(\infty)$ goes to 0 and for $(g - R(\infty))<0$, $R(\infty)$ goes to 1. Therefore, $R(\infty)$ lies between 0 and 1. To find nonzero solution for $R(\infty)$, $(g - R(\infty)) = 0$, $R(\infty)$ will be g.