To Randomize or Not To Randomize: Space Optimal Summaries for Hyperlink Analysis

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Personalized PageRank – Definition and Motivation

Definition: random surfer with *teleportation distribution* r and *tel. probab.* $c \approx 0.15$

$$\mathrm{PPR}_r(u) = c \cdot r(u) + (1-c) \sum_{v:(vu)\in E} \mathrm{PPR}_r(v)$$

Motivation: Search engines

- Improved ranking
- Fighting link spam

Slow to compute naively with the power method

Personalized PageRank – Linearity

Linearity:

$$\mathrm{PPR}_{\alpha_1 r_1+\alpha_2 r_2}(u) = \alpha_1 \mathrm{PPR}_{r_1}(u)+\alpha_2 \mathrm{PPR}_{r_2}(u)$$

Single page teleportation suffices:

$$\mathrm{PPR}_r(u) = \sum_v r(v) \cdot \mathrm{PPR}_v(u)$$

Personalized PageRank – Preliminaries

Two-phase algorithm

1. precomputes a PPR database
2. answers PageRank queries using the database

Exact PPR on a graph of $n \approx$ millions . . . billions of vertices:

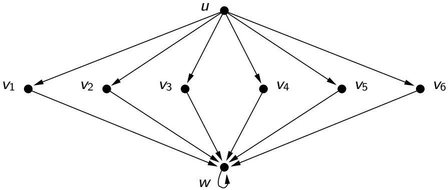
	Storage requirement	Person.
Topic sensitive [Haveliwala 02]	$O(t \cdot n)$ words	$t \approx 10 - 100$ topics
Hub decomp. [Jeh–Widom 03]	$O(h \cdot n)$ words	$h \approx 100.000$ pages
Lower bound of [Fogaras–Rácz 04]	$\Omega(n^2)$ bits, infeasible	all pages

Sampling Fully Personalized PageRank

- Express $\text{PPR}_u(v)$ as probability of random walk starting at u ending in v
- Sample ending points of random walks as above
- First algorithm with no restriction on u
- Additive error $\pm\epsilon$; out of bounds prob. δ
- Uses $O(n \cdot \epsilon^{-2} \log 1/\delta \log n)$ bits of space

Power Iteration and Dynamic Programming

Example



Power iteration amplifies the error downwards

Dynamic programming [Jeh–Widom WWW 2003] averages the error upward

$$\mathrm{PPR}_u^{(k+1)} = c\chi_u + (1-c) \cdot \sum_{v:(uv)\in E} \mathrm{PPR}_v^{(k)} / d^+(u)$$

Problem: small world, number of non-zeroes grow quickly in u's neighborhood

Rounded Dynamic Programming

Repeat $k_{\max} = 2 \log_{1-c} \epsilon$ times for all u

$$\widehat{\mathrm{PPR}}_u = \mathrm{Round}_k\Big(c\chi_u + (1-c) \cdot \sum_{v:(uv)\in E} \widehat{\mathrm{PPR}}_v / d^+(u)\Big)$$

- Space: n sparse PPR_u vectors in $O(n \cdot 1/\epsilon \log n)$ bits – optimal for top queries
- Can gradually decrease rounding error ϵ_k from $\epsilon_1 = 1$ to $\epsilon_{k_{\max}} = \epsilon$
- Deterministic output; inductive proof shows $\mathrm{PPR}_u(v) - 2\epsilon/c \leq \widehat{\mathrm{PPR}}_u(v) \leq \mathrm{PPR}_u(v)$
- Preprocessing: linear $O((n+m)/(c\epsilon))$ time

Dynamic Programming with Sketches

Drunken Surfer

- Mix up memories by random hash $h(v)$ of pages v

$$\mathrm{SPPR}_u(i) = \sum_{v:h(v)=i} \mathrm{PPR}_u(v) \quad \text{for } i = 1, \ldots, 2e/\epsilon$$

- Use surfers for $j = 1, \ldots, \log 1/\delta$ and use minimum vote: Count-Min Sketch [Cormode–Muthukrishnan 05]

$$\widehat{\mathrm{PPR}}_u(v) = \min_{j=1,\ldots,\log 1/\delta} \mathrm{SPPR}_u^{(j)}(h_j(v))$$

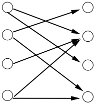
Dynamic Programming with Sketches Cont'd

- Dynamic programming over sketches by their linearity
- A variant also gives linear time preprocessing
- $O(n \cdot 1/\epsilon \log 1/\delta)$ bits of space – optimal for value queries

$$\mathrm{PPR}_u(v) - 2\epsilon/c - \epsilon \leq \widehat{\mathrm{PPR}}_u(v) \leq \mathrm{PPR}_u(v) + 2\epsilon/c$$

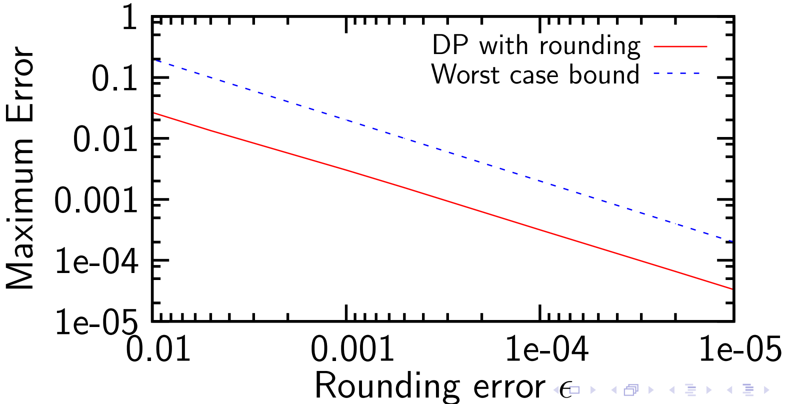
Lower Bounds

Reduction to one-way communication complexity of bit-vector probing

	Alice	Bob
	bit string $y \in \{0,1\}^{S}$	index i, output: y_i
1.	creates $G(y)$ 	
2.	transmits the PPR database of $G(y)$	
3.		queries the database for $\mathrm{PPR}_{u(i)}(v(i))$

Experiments

Stanford WebBase: $80M$ nodes, $800M$ edges
Measured accuracy over 1000 random nodes
Effect of rounding with $k_{\max} = 35$ iterations.



Quality of Approximate Rankings @ t

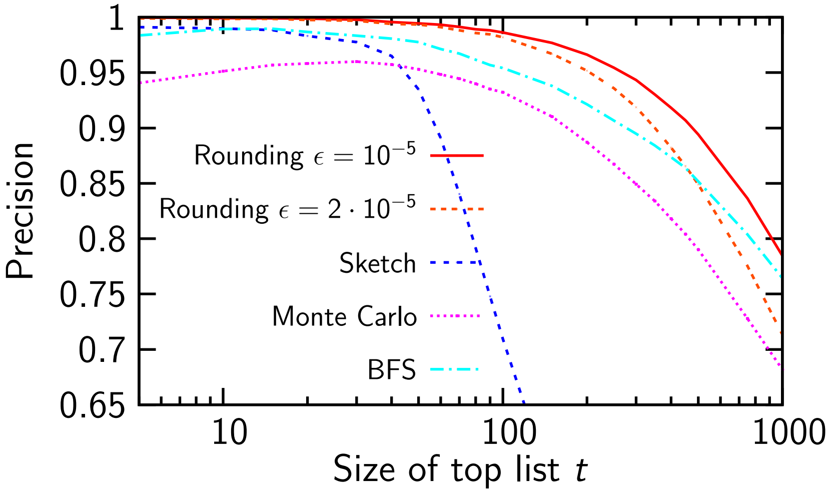
Precision = Recall:

$$\frac{|\text{approximate top-t} \cap \text{true top-t}|}{\text{t}}$$

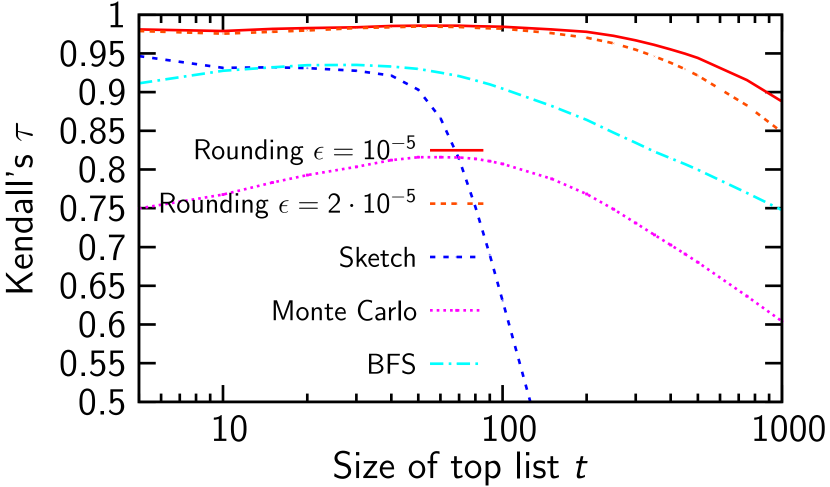
Kendall's Tau:

$$1 - 2\frac{\#\text{inversions in approximate top-t}}{\binom{\text{t}}{2}}$$

Precision


1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
Precision
10
100
1000
Size of top list t
Rounding $\epsilon = 10^{-5}$
Rounding $\epsilon = 2 \cdot 10^{-5}$
Sketch
Monte Carlo
BFS

Kendall's Tau



SimRank – Preliminaries and Sampling

"Two pages are similar if pointed to by similar pages" [Jeh–Widom 02]

$$\mathrm{Sim}^{(k)}(v_1, v_2) = \begin{cases} (1-c) \cdot \frac{\sum \mathrm{Sim}^{(k-1)}(u_1,u_2)}{d^-(v_1) \cdot d^-(v_2)} & \text{if } v_1 \neq v_2 \\ 1 & \text{if } v_1 = v_2. \end{cases}$$

$(1-c)^{k'}$-weighted path pair summation (incl. sampling [Fogaras–Rácz 05]) over

$$\begin{aligned} v_1 = w_0, w_1, \ldots, w_{k'-1}, w_{k'} &= u \\ v_2 = w'_0, w'_1, \ldots, w'_{k'-1}, w'_{k'} &= u \end{aligned}$$

SimRank – Reduction to Personalized PageRank

Version 0 reduction: count path pairs from v_1 and v_2 that may meet several times

$$\mathrm{Sim}^{(0)}_{v_1,v_2} = \sum_{k>0} (1-c)^k \sum_u \mathrm{RP}^{[k]}_{v_1}(u)\mathrm{RP}^{[k]}_{v_2}(u)$$

Recursively define self-similarity SimRank of *at least* $t+1$ inner meeting points as $\mathrm{SSim}^{(t+1)}(v)$

SimRank – Reduction to Personalized PageRank

Obtain SimRank by inclusion-exclusion of self-similarities

$$\mathrm{Sim}(v_1, v_2) = \sum_{k>0}(1-c)^k \sum_u \mathrm{RP}^{[k]}_{v_1}(u)\mathrm{RP}^{[k]}_{v_2}(u)\cdot\mathrm{SSim}(u)$$

$$\mathrm{SSim}(u) = 1 - \mathrm{SSim}^{(0)}(u) + \mathrm{SSim}^{(1)}(u) - \mathrm{SSim}^{(2)}(u) + \dots$$

Converges for $1 - c < 1/2$, technicalities to carry through approximation

Conclusion

- Efficient algorithms + lower bounds = space-optimal summaries for
  - Fully Personalized PageRank and for
  - SimRank with decay factor $< 1/2$
- At the heart of it: low space approximation of large vectors in the $\|\ldots\|_\infty$ norm
- Works well in practice

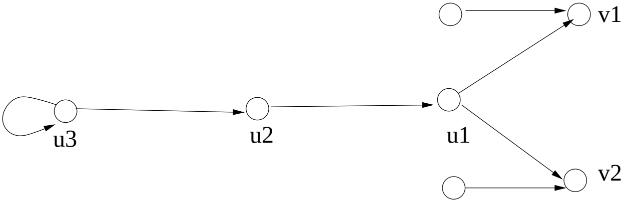
Thank you!

- http://www.ilab.sztaki.hu/websearch

Algorithms Compared

Algorithm	Running time
Dynamic Programming with $\epsilon = 2 \cdot 10^{-5}$ and $\epsilon = 10^{-5}$ rounding to varying ϵ_k	1.5 and 2.25 days
Dynamic Programming with $\epsilon = 6 \cdot 10^{-3}, \delta = 4 \cdot 10^{-3}$ sketches	6 days
Monte Carlo sampling with $N = 10000$ samples	6 days
Breadth First Search heuristic	3.5 days

SimRank Example



$$\sum_{k>0} \frac{1}{3^k} \sum_u \mathrm{RP}_{v_1}^{[k]}(u)\mathrm{RP}_{v_2}^{[k]}(u) = \frac{1}{4} \cdot \frac{1}{3} \left(1 + \frac{1}{3} + \frac{1}{3^2} + \dots\right) = \frac{1}{12} \cdot \frac{3}{2}$$

$$\mathrm{SSim}^{(0)}(u_i) = \frac{1}{3} + \frac{1}{3^2} + \dots = \frac{1}{2} \qquad \mathrm{SSim}^{(1)}(u_i) = \frac{1}{4}$$

$$\mathrm{SSim}(u_i) = 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots = \frac{2}{3} \checkmark$$