SPT-Based Topology Algorithm for Constructing Power Efficient Wireless Ad Hoc Networks

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ABSTRACT

In this paper, we present a localized Shortest Path Tree (SPT) based algorithm for constructing a sub-network with the minimum-energy property for a given wireless ad hoc network. Each mobile node determines its own transmission power based only on its local information. The proposed algorithm constructs local shortest path trees from the unit disk graph. The performance improvements of our algorithm are demonstrated through simulations.

Categories and Subject Descriptors

C.2.1 [**Network Architecture and Design**]: Network topology.

General Terms

Algorithms.

Keywords

Wireless ad hoc networks, topology control, power consumption.

1. INTRODUCTION

A wireless ad hoc network is formed from mobile nodes without using an infrastructure. Each mobile node is responsible for serving not only as a user but also as a router. The connectivity of the network is maintained through the cooperation of all nodes in the network. It is known that the performance of a protocol for an ad hoc network can be enhanced if the protocol is designed based on overlaying a virtual infrastructure over the ad hoc network. Also, due to the finite power supply of a mobile computer, power conservation has been widely used as a primary control parameter in the design of protocols for wireless ad hoc networks. Therefore, the problem of power-efficient topology control has been attracting more and more researchers from the areas of mobile computing and networking. A wireless ad hoc network can be modeled by a weighted directed graph $G = (V, E)$, where V represents the set of all mobile nodes and E represents the set of interconnections between pairs of mobile nodes. For each edge $(u, v) \in E$, node v must be in the transmission range of node u. We use $\|uv\|$ to denote the Euclidean distance between node u and node v. The weight of the edge (u, v), namely $w(u, v)$, can be formulated as $t \cdot \|uv\|^{\alpha} + rp(u, v)$ in the most widely-used power-attenuation model, where t is a threshold related to the signal-to-noise ratio at node u, α is a constant between 2.0 and 5.0 depending on the wireless transmission environment. The former part of the equation is typically called the *transmitter power* and is the power consumed for transmitting signal from node u to node v. The remaining part is the power consumed at the receiver node and is denoted as *receiver power*. The sum of transmitter power and receiver power is called *transmission power* in the rest of this paper.

We assume that all the mobile nodes are distributed in a two-dimensional plane and each mobile node has a GPS receiver on board for acquiring its own location information. We also assume that initially all mobile nodes are operated at full transmission power and have the transmission radius equal to one unit by a proper scaling. Consequently, the resulted graph G will be a unit-disk graph (denoted as UDG (V)) and there is an edge between two nodes if and only if their Euclidean distance is at most one. We assume that UDG (V) is strongly connected. All of the mobile nodes have unique identifiers (ID) numbered from 1 to N, where $N = |V|$. Each mobile node can individually adjust its own transmission power. We assume that omni-directional antennas are used by all of the mobile nodes to transmit and receive signals.

Hereafter we adopt several definitions given in [1]. Let f be a complete transmission power assignment on V, and G_f be the associated communication graph. The total power consumption of f is defined as $\Sigma_{u\in V} f(u)$, where $f(u)$ is the minimum transmission power needed to reach all the neighbors of u in G_f. Given a (unicast) path $\Pi(u, v)$ from node u to node v in G_f, the path can be expressed as $\Pi(u, v) = v_0v_1\ldots v_{h-1}v_h$, where $u = v_0$, $v = v_h$. The path length of $\Pi(u, v)$ (denoted as $|\Pi(u, v)|$) is h. The total transmission power of this path is defined as:

$$p(\Pi(u,v)) = \sum_{i=1}^{h} w(v_{i-1}, v_i).$$

Given a communication graph H, the *minimum-energy path* between node u and node v, denoted by $\Pi^{H}_{\min}(u, v)$, is a path whose total transmission power is the minimum among all the paths that connect these two nodes in H. A subgraph H of G has the minimum-energy property if for each $(u, v) \in V$ there is a minimum-energy path in H from u to v.

2. RELATED WORKS

In [2] Rodoplu and Meng described a distributed protocol for constructing a topology that guarantees the minimal-energy path between every pair of nodes that are connected in a given graph. Recently, Li *et al*. [3] proposed a protocol based on their results but performs better and is computationally simpler. In [1] Li *et al.* studied the power efficiency property of several well-known proximity graphs, such as the constrained Gabriel graph (denoted by GG (G)), the constrained relative neighborhood graph (denoted by RNG (G)), and the constrained Yao graph (denoted by YG_k (G)), over a (directed) graph G. They also showed that if G = UDG (V) and receiver power is negligible, the *power stretch factor* of GG (G) is always one, namely the minimum-energy property is guaranteed.

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WWW 2004, May 17–22, 2004, New York, New York, USA.
ACM 1-58113-912-8/04/0005.

The power stretch factors of RNG (G) and YG_k (G) could be as large as $N - 1$ and $1 / (1 - 2\sin(k/\pi))$, respectively.

3. OUR LOCALIZED ALGORITHM

Due to the characteristics of wireless ad hoc networks, it is preferred that the underlying network topology can be constructed in a localized manner [4]. In our case, a distributed transmission power control algorithm is called localized if every node u can decide its transmission power based only on the information of the nodes reachable in a small number of hops. The local topology view of node u, denoted by $LTV(u, k) = (V', E')$, is a subgraph of G such that (1) a node $v_i \in V'$ if the hop distance between v_i and u is no more than k; (2) an edge $(v_i, v_j) \in E'$ if $\|v_iv_j\|$ is less than the transmission radius of v_i.

Our localized algorithm operates based on $LTV(u, 1)$. The location information of the one-hop neighbors can be obtained by using some form of beacon messages that are sent periodically and asynchronously by each node; the weight of each edge in $LTV(u, 1)$ can thus be derived. Each node u applies Dijkstra's algorithm independently to get the shortest-paths from the source node u to the other nodes in $LTV(u, 1)$. As a result, the local shortest path tree of node u, denoted by $LSPT(u)$, can be obtained. The direct children of node u, $DC(u)$, is defined as $DC(u) = \{v \in V' \mid h(LSPT(u), v) = 1\}$, where $h(LSPT(u), v)$ is the height of a child node v in $LSPT(u)$. Node u then removes the edge set $\{(u, w) \mid w \notin DC(u)\}$ from its edges and decides its logical links. Each node sends the result to its one-hop neighbors. The topology generated under the above descriptions is denoted as G^1. Since for each node u, only the one-hop neighborhood information is available for constructing $LSPT(u)$, some links in G^1 may be uni-directional. However, uni-directional links are unfavorable in wireless ad hoc networks. Our solution to remove uni-directional links is simple: since all nodes are aware of the logical links of its one-hop neighbors after the above process is completed, each node deletes the uni-directional links and adjusts its transmission radius according to the remaining logical links. The resulted topology is denoted as G^2. An example is illustrated in Fig. 1; the gray circle is the initial transmission range of p_1 and the dashed circle represents the transmission range after adjustment.

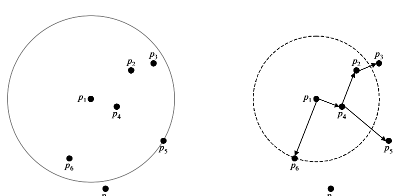

Fig 1. Illustrative example.

The critical properties of the topology generated by our algorithm are listed below. Due to the limited space, we omit the proofs here.

***Lemma 1**:* The minimum-energy path between any two nodes in G is preserved in G^1.

***Lemma 2**:* The minimum-energy path between the two end nodes of each deleted link in G^1 is preserved in G^2.

***Lemma 3**:* G^2 preserves the network connectivity of G.

4. PERFORMANCE COMPARISONS

Via simulations, we compared the performance of our algorithms with that of others in terms of the total power consumption (denoted as *tpc*) and the average/max node degree (denoted as *avg*/*max nd*) of the topologies constructed by the algorithms being compared. The experimental results are summarized in Table 1. Our algorithm is denoted as LSPT. The unit disk graph (denoted as UDG) is chosen as a basis for comparison. We chose the algorithm proposed by Li and Halpern [3] (denoted by SMECN) since it performs significantly better than the one proposed by Rodoplu and Meng [2] in terms of total power consumption. Likewise, the constrained Gabriel graph (denoted by GG) has the minimum-energy property and outperforms those described in [1]; it is thus chosen for comparison here. In our simulations, α was set to 4.0 and the receiver power was ignored. Be advised that *tpc* is normalized to lie in between 0.0 and 1.0 by dividing its values by the total power consumption of UDG. The transmitter range R is fixed at 500 meters. The map sizes are equal to $s \times R$ by $s \times R$, for $s = 3$ and 5. The x and y coordinates of each node are selected at random in the interval $[0, m]$, where m is the map size. The experimentation was performed for $N = 100$.

Table 1. The performance measurements

	$s = 3$			$s = 5$		
	tpc	*avg nd*	*max nd*	*tpc*	*avg nd*	*max nd*
UDG	1.0	25.5408	53	1.0	10.4232	27
GG	0.05339	3.59	8	0.245597	3.4496	8
SMECN	0.026874	2.7016	6	0.159539	2.6596	5
LSPT	0.01495	2.4284	5	0.122629	2.45	5

From Table 1 we observe that the topology constructed by our algorithm has a *tpc* much less than that of UDG, GG and SMECN. Our algorithm also outperforms the others in terms of average and max node degree.

5. CONCLUSIONS

In this paper, we develop a distributed algorithm that requires only local information for constructing a logical topology on the given unit disk graph. The concept of k-redundant edges is proposed by Li and Halpern [3]. The algorithm in [3], however, comes with 2-redundant edges only. Our proposed algorithm tackles k-redundant edges for $k \geq 2$. That is, we achieve a better result by a means that is totally different from those given in [2, 3]. Moreover, the topology constructed by our algorithm has several desired features such as low total power consumption and the minimum-energy property.

6. REFERENCES

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